HIGHER ORDER HOMOGENOUS DIFFERENTIAL EQUATIONS

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Solve the following differential equation:

$$y'' - 4y' + 4y = 0 \qquad y(0) = 1 \qquad y'(0) = 3$$

Problem 2:

Solve the following differential equation:

$$y'' - 2y' + 4y = 0$$

Problem 3:

Solve the following differential equation:

$$y'' - 6y' + 9y = 0$$

HIGHER ORDER HOMOGENOUS DIFFERENTIAL EQUATIONS

Solution 1:

The first step is to put the equation in to its characteristic form simply by adding an r variable in the place of the y variables. The characteristic equation is:

$r^2 - 4r + 4 = 0$ which factors out to $(r-2)(r-2) = 0$

The roots are both r=2, which shows that we have two equal roots, making the general solution:

$$y_h = c_1 e^{2t} + c_2 t e^{2t}$$

Using the initial conditions, determine the unknown coefficients:

$y(0) = 1 = c_1 e^{2(0)} + c_2 (0) e^{2(0)} \Rightarrow c_1 = 1$ so

$$y_h = e^{2t} + c_2 t e^{2t}$$

$$y_h' = 2e^{2t} + 2c_2 t e^{2t} + c_2 e^{2t}$$

$$y'(0) = 3 = 2e^{2(0)} + 2c_2 (0) e^{2(0)} + c_2 e^{2(0)} \Rightarrow c_2 = 1$$

Plug these values back in to the general solution so that:

$$y_h = e^{2t} + t e^{2t}$$

Solution 2:

The first step is to put the equation in to its characteristic form simply by adding an r variable in the place of the y variables. The characteristic equation is:

$r^2 - 2r + 4 = 0$ which does not factor so use the quadratic formula.

With a=1, b=-2, and c=4 the roots are:

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)} = 1 \pm j\sqrt{3}$$

The roots are $1 \pm j\sqrt{3}$, which shows that we have two complex roots, making the general solution:

$$y(t) = e^{\alpha x}(c_1 \cos \omega x + c_2 \sin \omega x) \text{ where } \alpha = 1 \text{ and } \omega = \sqrt{3}$$
$$y(t) = e^{x}(c_1 \cos \sqrt{3} x + c_2 \sin \sqrt{3} x)$$

If initial conditions were given we could determine the unknown coefficients from here.

Solution 3:

The first step is to put the equation in to its characteristic form simply by adding an r variable in the place of the y variables. The characteristic equation is:

$$r^2 - 6r + 9 = 0 \text{ which factors out to } (r-3)(r-3) = 0$$

The roots are both r=3, which shows that we have two equal roots, making the general solution:

$$y_h = c_1 e^{3t} + c_2 t e^{3t}$$

If initial conditions were given we could determine the unknown coefficients from here.