INTEGRATION BY PARTS

A common mistake one makes when integrating a product of two functions is to simplify the function in to the product of the integrals of the individual functions, such as:

$$\int f(x)g(x)dx = \int f(x)dx \int g(x)dx$$

To complete the integration of the product of two functions, the correct procedure is to use the Integration by Parts. To fully understand the process, recall first that the product rule states:

$$[f(x)g(x)]' = f(x)g'(x) + g(x)f'(x)$$

Integrating both sides gives:

$$\int [f(x)g(x)]'dx = \int f(x)g'(x) + g(x)f'(x)dx$$
$$f(x)g(x) = \int f(x)g'(x)dx + \int g(x)f'(x)dx$$

Rearranging the equation:

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$

This is the general formula when dealing with the integration by parts. Typically it is not written in this form, but rather, by using the following notation:

Let: $u = f(x)$ and $v = g(x)$
$du = f'(x)$ and $dv = g'(x)$

So that the general equation becomes:

$$udv = uv - \int vdu$$

With this general equation in mind, the process of integration of parts follows four steps:

1. Write the given integral

$$\int f(x)g(x)dx$$

2. Define the intermediary functions u and v and there derivatives such that:

$$u = f(x) \qquad dv = g(x)dx$$
$$du = f'(x) \quad v = \int g(x)dx$$

3. Plug the values in to the general formula:

$$udv = uv - \int vdu$$

4. Complete the new integral on the right side of the equation:

$$\int vdu$$

Both indefinite and definite integrals can be solved using the Integration by parts.

Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

Integrate $\int xe^x dx$

Solution:

Integrate using the four steps of integration by substitution:

1. Write the given integral

$$\int xe^x dx$$

2. Define the intermediary functions u and v and there derivatives such that:

$$u = x \qquad dv = e^x dx$$

$$du = dx \qquad v = e^x.$$

3. Plug the values in to the general formula:

$$\int u\,dv = uv - \int v\,du$$
$$\int xe^x = xe^x - \int e^x dx$$

4. Complete the new integral on the right side of the equation:

$$\int e^x dx = e^x + c$$

Therefore, $\int xe^x dx = xe^x - e^x + c$