# L'HOPITAL'S RULE

Consider the limit of the function:

$$\lim_{x\to a}\frac{f(x)}{g(x)}$$

If both the numerator and the denominator are finite at a and $g(x)\neq 0$, then:

$$\lim_{x\to a}\frac{f(a)}{g(a)}$$

However, when the limit of the function is an indeterminate form, the L'Hopital's Rule is employed.

Indeterminate forms we often encounter are:

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0}$$
$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\infty}{\infty}$$

L'Hopital's Rule provides a method to evaluating the limits when these forms are met.

**L'Hopital's Rule for $\frac{0}{0}$**

If the $\lim f(x)=\lim g(x)=0$, then:

- If $\lim\frac{f'(x)}{g'(x)}=L$, then $\lim\frac{f(x)}{g(x)}=\lim\frac{f'(x)}{g'(x)}+L$
- If $\lim\frac{f'(x)}{g'(x)}$ tends to $+\infty$ or $-\infty$, then so does $\frac{f(x)}{g(x)}$

**L'Hopital's Rule for $\frac{\infty}{\infty}$**

If the $\lim f(x)$ and $\lim g(x)$ are both infinite, then:

- If $\lim \dfrac{f'(x)}{g'(x)} = L$, then $\lim \dfrac{f(x)}{g(x)} = \lim \dfrac{f'(x)}{g'(x)} + L$
- If $\lim \dfrac{f'(x)}{g'(x)}$ tends to $+\infty$ or $-\infty$, then so does $\dfrac{f(x)}{g(x)}$

It's important to note that L'Hopitals Rule only applies to indeterminate forms. If it is applied to a function that can be determined without using the rule, then an incorrect result will be obtained.

Also, if an indeterminate form is obtained after L'Hopital's Rule is used once, then it can be used again on that limit until a limit is determined.

## Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

Evaluate $\lim\limits_{x \to 2} \dfrac{x^2 - 4x + 4}{x - 2}$

## Solution:

Using normal procedures and taking the limit of the function results in an indeterminate form $\frac{0}{0}$.

$$\lim_{x \to 2} \frac{x^2 - 4x + 4}{x - 2} = \frac{2^2 - 4(2) + 4}{2 - 2} = \frac{0}{0}$$

However, L' Hôpital's Rule can be used to assist in determining the actual limit, recall that:

If the $\lim f(x) = \lim g(x) = 0$, then:

- If $\lim \frac{f'(x)}{g'(x)} = L$, then $\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)} + L$
- If $\lim \frac{f'(x)}{g'(x)}$ tends to $+\infty$ or $-\infty$, then so does $\frac{f(x)}{g(x)}$

Therefore, carrying out L' Hôpital's Rule gives:

$$\lim_{x \to 2} \frac{x^2 - 4x + 4}{x - 2} = \lim_{x \to 2} \frac{2x - 4}{1} = \lim_{x \to 2} \frac{2(2) - 4}{1} = 0$$