CURVATURE

The curvature measures how fast the curve of a function is changing direction at any given point.

Given a curve defined by the equation $y = f(x)$, the curvature at any point of the curve can be found using the following equation:

$$k = \frac{\left|f''(x)\right|}{\left(1+\left[f'(x)\right]^2\right)^{\frac{3}{2}}}$$

When designing public systems, we may need to ensure the curvature of the system (roadway, train track, etc) will be safe for a provided specified use. In this case, in may be necessary to determine the radius of curvature of the system, which is defined as the radius of the approximating circle. This radius changes as an objects move along the curve and can be found using the following formula:

$$\text{Radius of Curvature, } \rho = \frac{\left(1+\left[f'(x)\right]^2\right)^{\frac{3}{2}}}{\left|f''(x)\right|} = \frac{1}{k}$$

Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

Determine the radius of curvature for the function $f(x) = 2x^3 - x + 3$ at $x = 1$

Solution:

Now, to find the radius of curvature, we first need to determine the derivative of f(x) which is:

$$f'(x) = 6x^2 - 1$$

And

$$[f'(x)]^2 = (6x^2 - 1)^2 = 36x^4 - 12x^2 + 1$$

Determine the Second Derivative:

$$f''(x) = 12x$$

Now substitute these values in to the formula to determine the radius of curvature at at any point x:

$$\rho = \frac{\left(1 + [f'(x)]^2\right)^{\frac{3}{2}}}{|f''(x)|}$$

$$= \frac{\left[1 + 36x^4 - 12x^2 + 1\right]^{\frac{3}{2}}}{|12x|}$$

$$= \frac{\left[36x^4 - 12x^2 + 2\right]^{\frac{3}{2}}}{|12x|}$$

Now to determine the radius of curvature at the required point x=1, plug the value in so that:

$$\left[\frac{\left[36(1)^4 - 12(1)^2 + 2\right]^{3/2}}{|12(1)|}\right]_{x=1} = 11.05$$