CONFIDENCE INTERVALS

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

In a study, the average time that 22 college students watched TV during the week was 19.6 hours with a standard deviation of 5.8 hours. Construct a 90% confidence interval assuming the data is normally distributed.

Problem 2:

A random sample of 30 students has a mean annual earnings of $3,120 with a standard deviation of $677. Determine the 90% confidence interval.

Problem 3:

A sample of 100 vegetable cans showed an average weight of 13 ounces with a standard deviation of .8 ounces. Determine the 90% confidence interval of the population.

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Solution 1:

There are four steps to defining a confidence interval.

- Identify a sample statistic that will be used to estimate a population parameter, usually a mean or proportion.

  This problem deals with a sample mean, $\bar{x} = 19.6$

- Select a confidence level describing the uncertainty of the sampling method.

  The problem requests a 90% confidence interval

- Find the margin of error calculated using the following equation:

  Determine the Critical value:

  - Compute alpha, α = 1 - (90 / 100)=.1
  - Find the critical probability, $p^* = 1 - \frac{.1}{2} = .95$
  - Express the critical value as a t score by:
    - Finding the degrees of freedom (DF), 22-1=21
    - The critical t score, t^*, is the t score having degrees of freedom equal to DF and a cumulative probability equal to the critical probability, p^*, 1.721

  Determine the Standard Error:

$$SE = \frac{5.8}{\sqrt{22}} = 1.24$$

The Margin of error (ME) = 1.721(1.25)=2.15

- The 90% confidence interval is then 19.6 ± 2.15

It's important to note that this problem could have been done using the z-score along with the Normal Distribution tables, but would result in a slightly different answer and a tighter interval. However, when a sample size is small (below 30) it is more appropriate to use the t-score and the t-distribution tables.

Solution 2:

There are four steps to defining a confidence interval.

- Identify a sample statistic that will be used to estimate a population parameter, usually a mean or proportion.

  This problem deals with a sample mean, $\overline{x} = 3{,}120$

- Select a confidence level describing the uncertainty of the sampling method.

  The problem requests a 90% confidence interval

- Find the margin of error calculated using the following equation:

  Determine the Critical value:

  - Compute alpha, α = 1 - (90 / 100)=.1
  - Find the critical probability, $p^* = 1 - \frac{.1}{2} = .95$
  - Express the critical value as a t score by:
    - Finding the degrees of freedom (DF), 30-1=29
    - The critical t score, t^*, is the t score having degrees of freedom equal to DF and a cumulative probability equal to the critical probability, p^*, 1.699

Determine the Standard Error:

$$SE = \frac{677}{\sqrt{30}} = 123.6$$

The Margin of error (ME) = 1.699(123.6)=210

- The 90% confidence interval is then $3,120 \pm 210$

It's important to note that this problem could have been done using the z-score along with the Normal Distribution tables, but would result in a slightly different answer and a tighter interval. However, when a sample size is small (below 30) it is more appropriate to use the t-score and the t-distribution tables.

Solution 3:

There are four steps to defining a confidence interval.

- Identify a sample statistic that will be used to estimate a population parameter, usually a mean or proportion.

  This problem deals with a sample mean, $\bar{x} = 13$

- Select a confidence level describing the uncertainty of the sampling method.

  The problem requests a 90% confidence interval

- Find the margin of error calculated using the following equation:

  Determine the Critical value:

  - Compute alpha, α = 1 - (90 / 100)=.1

- Find the critical probability, $p^* = 1 - \frac{.1}{2} = .95$
- Express the critical value as a t score by:
  - Finding the degrees of freedom (DF), 100-1=99
  - The critical t score, t^*, is the t score having degrees of freedom equal to DF and a cumulative probability equal to the critical probability, p^*, 1.660

Determine the Standard Error:

$$SE = \frac{.8}{\sqrt{100}} = .08$$

The Margin of error (ME) = 1.660(.08)=.133

- The 90% confidence interval is then $13 \pm .133$

It's important to note that this problem could have been done using the z-score along with the Normal Distribution tables, but would result in a slightly different answer and a tighter interval. We used the t-score, which will yield as close of an answer when the DF is as high as it was in this question.