T DISTRIBUTIONS

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

An engineer completed a study and recorded the failure strengths of 12 concrete columns used as support for a building. The standard deviation of the samples was 989 kN. From previous tests, it is known that the failure strengths of these types of columns are normally distributed with a mean of 3000 kN. What is the probability of the mean failure strength of these 12 samples is lower than 2000 kN.

Problem 2:

The pH levels of a certain stream are often tested by taking 12 samples from different parts of the stream. It is known from past experiments that the pH of the water is approximately normally distributed. Determine the probability that the sample mean of the latest 12 pH measurements will be within .2 units of the true average pH for the entire stream. The most recent measurements were:

6.63	6.59	6.65	6.67	6.54	6.13
6.62	7.13	6.68	6.82	7.62	6.56

Problem 3:

A certain University board administered an exam to 25 randomly selected students. They found that the average score was 115 with a standard deviation of 11. If we can assume that the cumulative probability is 0.90, what population mean would have produced this sample result?

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Solution 1:

The approach to solving this problem follows a two-step process as follows:

- Compute a t score, assuming that the mean of the sample test is 2000 kN.
- Determine the cumulative probability for that t score.

The first step involves calculating the t score using the following equation:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Where $\bar{x}$ is the sample mean (2000), μ is the population mean (3000), s is the standard deviation of the sample (989), and n is the sample size (12). Plugging these values in to the equation we get:

$$t = \frac{2000 - 3000}{\frac{989}{\sqrt{12}}} = -3.50$$

The next step is to determining the cumulative probability for the t score using the t distribution tables. We know:

- $t = -3.50$
- $DF = 12 - 1 = 11$

Using the t distribution tables we find that these values correspond to a cumulative probability of .0025. Therefore, there is a .25% chance that the mean failure strength of these 12 samples is lower than 2000 kN.

Solution 2:

The approach to solving this problem is the same as before:

- Compute a t score, this time assuming that $\overline{x}-\mu=.2$ (that the sample mean of the latest measurements will be within .2 units of the true average)
- Determine the cumulative probability for that t score

The first step involves calculating the t score using the following equation:

$$t=\frac{\overline{x}-\mu}{\frac{s}{\sqrt{n}}}$$

Where $\overline{x}-\mu=.2$, s is the standard deviation of the sample (.362), and n is the sample size (12). Plugging these values in to the equation we get:

$$t=\frac{.2}{\frac{.362}{\sqrt{12}}}=1.91$$

The next step is to determining the cumulative probability for the t score using the t distribution tables. We know:

- $t=1.91$
- $DF=12-1=11$

Using the t distribution tables we find that these values correspond to a cumulative probability of approximately .04. The samples can be within .02 units on either side of the true mean, so we multiply .04 by 2, which is .08. Therefore, there is an 8% chance that the mean of the latest 12 pH measurements are not within .2 units, or that there is a 92% chance that they are within .2 units.

Solution 3:

In this problem, we are given a cumulative probability equal to .90 and asked to determine the population mean. The process of solving is:

- Determine the t score based on the cumulative probability and degrees of freedom.
- Use the general equation for a t score to solve for the population mean.

Using the t distribution tables, we find that a cumulative probability of .9 with 24 degrees of freedom corresponds to a t score of 1.318.

The next step involves calculating the population mean using the following equation:

$$t = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Where $\overline{x}$ is the sample mean (115), μ is the population mean (unknown), s is the standard deviation of the sample (11), n is the sample size (25), and the t score is 1.318. Plugging these values in to the equation we get:

$$1.318 = \frac{115 - \mu}{\frac{11}{\sqrt{25}}} \Rightarrow \mu = 112.1$$

Therefore, a population mean of 112.1 would have produced a cumulative probability of .9.