BINOMIAL DISTRIBUTION AND PROBABILITY

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Of all the cars registered in US, 53% are made in the US. In a sample of 12 cars registered in the United States, what is the probability that 9 are foreign made?

Problem 2:

A research team at a certain University conducted a study showing that approximately 10% of all businessmen who wear ties wear them so tight that they actually reduce blood flow to the brain. At a meeting of 20 businessmen, all of whom were wearing ties, what is the probability that at least one tie is too tight?

Problem 3:

A reputable magazine states that approximately 70% of all people who buy prescription glasses from a private doctor's office were highly satisfied. In a sample of 11 people buying prescription glasses from a private doctor, what is the probability that less than 10 are highly satisfied?

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Solution 1:

The problem gives the percentage of "US made" cars, but then asks for the percentage of "foreign made" cars. If we want to go with the 9 cars foreign made we must first calculate the probability that a car is foreign made. This is of course 1-(% of US made cars) = 47%. Therefore:

$$\text{P(9 are foreign made)} = {}_{12}C_9 \cdot .47^9 \cdot (1-.47)^3 = .037$$

Solution 2:

At least 1 means 1 or more. One way of doing this problem would be to calculate the probability, using the binomial formula, that 1 tie is too tight, 2 ties are too tight, 3 ties are too tight, and so on, up to and including 20 ties are too tight. These probabilities would then be added up to get an answer to the question. This is obviously a lot of work that can be avoided. The answer can be determined by knowing that a fact of probability is:

$$P(0)+P(1)+P(2)+...+P(19)+P(20)=1$$

This can be written as:

$$P(0)+P(at\ least\ 1)=1$$

After subtracting P(0) from both sides of the equation, we get:

$$\text{P(at least 1 tie is too tight)} = 1 - \text{P(0 ties are too tight)}$$

$$\text{P(at least 1 tie is too tight)} = 1 - {}_{20}C_0 \cdot .1^0 \cdot (1-.1)^{20} = 1 - .122 = .878$$

Solution 3:

Since less than 10 means 9 or 8 or 7 or 6, all the way down to 0, one way of doing this problem would be to calculate the probability that 9 people are highly satisfied, 8 people are highly satisfied, all the way down to 0 people being highly satisfied, and then add up all these probabilities. Once again, a quicker way to determine the answer is by using the fact that:

$$P(0)+P(1)+P(2)+....+P(9)+P(10)+P(11)=1$$

This can be written as:

$$P(\textit{less than } 10)+P(10)+P(11)=1$$

After subtracting P(10) and P(11) from both sides we will get:

$$P(\textit{less than } 10 \textit{ people satisfied})=1-P(10)-P(11)$$
$$P(\textit{less than } 10 \textit{ people satisfied})=1-{}_{11}C_{10}\cdot.7^{10}\cdot(1-.7)^{1}-{}_{11}C_{11}\cdot.7^{11}\cdot(1-.7)^{0}$$
$$P(\textit{less than } 10 \textit{ people satisfied})=1-.093-.02=.887$$