MEASURES OF DISPERSION

When discussing Measures of Dispersion, we are more specifically talking about the variance and standard deviation of the spread of data within a population or sample.

It is important to distinguish between the variance of a population and the variance of a sample. They have different notation, and they are computed differently. The variance of a population is denoted by σ^2; and the variance of a sample, by s^2.

The population variance, σ^2 , and standard deviation, σ, are the deviation among individual measurements from a population mean, for the entire population.

As is the case with the mean, the population variance and standard deviation are the expected deviations. The population variance is calculated using the population mean and is found using the general formula:

$$\sigma^2 = \frac{1}{n}\sum(x_i - \mu)^2$$

The population standard deviation is simply the square-root of the population variance.

The variance of a sample is defined slightly different.

The sample variance, s^2, and standard deviation, S, is how much each individual measurement deviates from the sample mean and is found using the general formula:

$$s^2 = \frac{1}{n-1}\sum(x_i - \overline{x})^2$$

To calculate the sample variance, we first find the errors of all the measurements, that is, the difference between each measurement and the sample mean, $(x_i - \overline{x})$. We then square each value and add them all together, then divide by the number of samples minus 1.

The sample standard deviation is simply the square-root of the sample variance, $\sqrt{s^2}$.

Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

Suppose that the following table contains the total number of hours students spent studying the week leading up to a certain Engineering exam.

28.4	44.4	36.4	33.5
34.9	30.5	32.7	24.6

Determine the population variance if the population mean is 33.18

Solution:

The population variance is calculated using the population mean and is found using the general formula:

$$\sigma^2 = \frac{1}{n}\sum(x_i - \mu)^2$$

The population mean is known to be 33.8 and the number of measurements is 8.

Now we need to determine the sum of the $(x_i - \mu)^2$ terms. To do this, simply use a table:

x_i	$x_i - \mu$	$(x_i - \mu)^2$
28.4	-4.78	22.85
44.4	11.22	125.89

 psst…don't forget to take notes

36.4	3.22	10.37
33.5	.32	.10
34.9	1.72	2.96
30.5	-2.68	7.18
32.7	-.48	.23
24.6	-8.58	73.62
		$\sum(x_i-\mu)^2 = 243.195$

Therefore, plugging the values, the variance of this population is:

$$\sigma^2 = \frac{243.20}{8} = 30.4$$