FUNDAMENTAL LAWS OF PROBABILITY

Using the laws of probability will allow one to determine the probability of an event from the known probabilities of other events.

Some important terms should be noted up front:

- When talking events, two events are mutually exclusive if they can't occur at the same time.
- That probability that Event A occurs, given that Event B has already occurred, is called the conditional probability. The conditional probability of Event A, given Event B, is denoted as $P(A|B)$
- The complement of an event is the event not occurring. The probability of an event not occurring is written as $P(A')$
- The probability that events A and B both occur is the probability of the intersection of A and B. The probability of the intersection of events A and B is written as $P(A \cap B)$. If events A and B are mutually exclusive, $P(A \cap B) = 0$
- The probability that Events A or B occur is the probability of the union of A and B, written as $P(A \cup B)$
- If the occurrence of event A changes the probability of Event B, then events A and B are dependent. On the other hand, if the occurrence of Event A does not change the probability of Event B, then Events A and B are independent.
- The probability of an event occurring ranges from 0 to 1
- The sum of probabilities of all possible events is always equal 1

Some of the most used Laws of Probability are:

Rule of Subtraction:

The probability that event A will occur is equal to 1 minus the probability that event A will not occur, that is to say:

$$P(A) = 1 - P(A')$$

Rule of Multiplication:

When we want to know the probability that two events (Event A and Event B) both occur, we are dealing with a situation of the intersection of two events. The rule of multiplication says that the probability that Events A and B both occur is equal to the probability that Event A occurs times the probability that Event B occurs, given that A has occurred, in others words:

$$P(A \cap B) = P(A)P(B \mid A)$$

Rule of Addition:

When we have two events, and we want to know if either of the events will occur, the rule of addition is used. The probability that event A or event B occurs is equal to the probability of Event A occurs plus the probability that event B occurs minus the probability that both event A and B occur:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Where $P(A \cap B)$ is defined above as being $P(A)P(B \mid A)$, so this rule can be rewritten as:

$$P(A \cup B) = P(A) + P(B) - P(A)P(B \mid A)$$

Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

Determine the probabilities of selecting at random a:

A) College student

B) Professor

From a crowd containing 20 college students and 33 professors.

Solution:

The probability of selecting a college student at random out of the crowd is given by:

$$p = \frac{number\ of\ college\ students}{number\ in\ crowd}$$

$$p = \frac{20}{20+33} = \frac{20}{53}$$

There is a 37.7% chance of selecting a college student at random out of the defined crowd.

The probability of selecting a professor at random out of the crowd is given by:

$$p = \frac{number\ of\ professors}{number\ in\ crowd}$$

$$p = \frac{33}{20+33} = \frac{33}{53}$$

There is a 62.3% chance of selecting a professor at random out of the defined crowd.

It’s important to note that total probability always adds up to 1.

In this case:

$$\frac{20}{53} + \frac{33}{53} = 1$$