FUNDAMENTAL LAWS OF PROBABILITY

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Determine the probabilities of selecting at random:

A) The winning car in a race in which includes 10 cars
B) The winning car in both the first and second race if each race includes 10 cars

Problem 2:

The probability of a propietary component failing over it's lifetime due to excessive temperature is $\frac{1}{20}$, due to excessive vibration is $\frac{1}{25}$, and due to excessive humidity is $\frac{1}{50}$. Determine the probabilities that during the lifetime of the component, it will:

A) Fail due to excessive temperature and excessive vibration

B) Fail due to excessive vibration or excessive humidity

C) Not fail because of both excessive temperature and excessive humidity

Problem 3:

A batch of 100 components contains 73 what are withing the required tolerance values, 17 which are below the required tolerance values, and the remainder are above the required tolerance values. Determine the probabilities that:

A) When randomly selecting a component and then a second component, that both are within the required tolerance when selecting with replacement.
B) The first component chosen is below and the second component chosen is above the required tolerances, when the selection is without replacement.

FUNDAMENTAL LAWS OF PROBABILITY

Solution 1:

A) Since only one of the ten cars can win, the probability of selecting at random the winning car is:

$$p = \frac{number\ of\ winners}{number\ of\ cars}$$

$$p = \frac{1}{10} = .10$$

There is a 10% chance of selecting the winning car at random out of the entire field.

B) The probability of selecting the winning car in the first race is $\frac{1}{10}$. The probability of selecting the winning car in the second race is $\frac{1}{10}$ as well. The probability of selecting the winning car in both the first and second race is given by the multiplication law of probability or:

$$p = p_1 \cdot p_2$$

$$p = \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100} = .01$$

There is a 1% chance of selecting the winning car at random in both the first and second race.

Solution 2:

First define all the possible individual probabilities:

Let p_t be the probability of failure due to excessive temperature, then:

$$p_t = \frac{1}{20}$$

Let p_t' represent the probability that it will not fail due to excessive temperature, then:

$$p_t' = \frac{19}{20}$$

Let p_v be the probability of failure due to excessive vibration, then:

$$p_v = \frac{1}{25}$$

Let p_v' represent the probability that it will not fail due to excessive vibration, then:

$$p_v' = \frac{24}{25}$$

Let p_h be the probability of failure due to excessive vibration, then:

$$p_h = \frac{1}{50}$$

Let p_h' represent the probability that it will not fail due to excessive vibration, then:

$$p_h' = \frac{49}{50}$$

A) The probability that the component will Fail due to excessive temperature and excessive vibration is:

$$p_t \cdot p_v = \frac{1}{20} \cdot \frac{1}{25} = \frac{1}{500} = .002$$

There is a .2% chance that the component will Fail due to excessive temperature and excessive vibration

B) The probability that the component will Fail due to excessive vibration or excessive humidity is:

$$p_v + p_h = \frac{1}{25} + \frac{1}{50} = \frac{3}{50} = .06$$

There is a 6 % chance that the component will Fail due to excessive vibration or excessive humidity

C) The probability that the component will not Fail because of both excessive temperature and excessive humidity

$$p_t' \cdot p_h' = \frac{19}{20} \cdot \frac{49}{50} = \frac{931}{1000} = .931$$

There is a 93.1 % chance that the component will not Fail because of both excessive temperature and excessive humidity

Solution 3:

A) The probability of selecting a component within the required tolerance values is $\frac{73}{100}$. The first component drawn is now replaced and a second one is drawn from the batch of 100. The probability that this component will be within the required tolerances is $\frac{73}{100}$.

Therefore, the probability of selecting a component withing the required tolerances for both the first and second draw is:

$$p = \frac{73}{100} \cdot \frac{73}{100} = \frac{5329}{10000} = .5329$$

B) The probability of obtaining a component below the required tolerance values on the first draw is $\frac{17}{100}$. There are now only 99 components left to choose from since we are drawing without replacement. Therefore, the probability of selecting a component on the second draw that is above the required tolerances is $\frac{10}{99}$.

Therefore, the probability of randomly selecting a component below the required tolerance values, followed by selecting a component above the required tolerances is:

$$\frac{17}{100}\cdot\frac{10}{99}=\frac{170}{9900}=.0172$$