TAYLOR AND MACLAURIN SERIES

Provided a power series representation for the function $f(x)$ about $x = a$ exists, the Taylor Series for $f(x)$ about $x = a$ is:

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{n}(a)}{n!}(x-a)^{n}=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^{2}+\frac{f'''(a)}{3!}(x-a)^{3}+\ldots$$

If we consider a Taylor series about $x = 0$, so $a = 0$, the series is called a Maclaurin Series for $f(x)$, generally represented as:

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{n}(0)}{n!}x^{n}=f(0)+f'(0)x+\frac{f''(0)}{2!}x^{2}+\frac{f'''(0)}{3!}x^{3}+\ldots$$

To find the Taylor Series for a function the general formula for $f^{n}(a)$ must be defined and then plugged in to the general formula of the Taylor Series.

The first step in finding this general formula, $f^{n}(a)$, is to take a number of derivatives of the function $f(x)$, followed up by evaluating those derivatives at $x = a$. After a few computations, there will be a clear pattern to the evaluations that can then be used to define the general formula that will be plugged in to the Taylor Series.

Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

Determine the Taylor Series about x=-1 for $f(x)=\frac{1}{x}$

Solution:

Recall that the Taylor Series for $f(x)$ about $x=a$ is:

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{n}(a)}{n!}(x-a)^{n}=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^{2}+\frac{f'''(a)}{3!}(x-a)^{3}+\ldots$$

To find the Taylor Series for a function the general formula for $f^{n}(a)$ must be defined and then plugged in to the general formula of the Taylor Series. The general formula $f^{n}(a)$ can be derived as:

$$f(a)=x^{-1} \qquad f(-1)=-1$$

$$f'(a)=-x^{-2} \qquad f'(-1)=-1$$

$$f''(a)=2x^{-3} \qquad f''(-1)=-2$$

$$f'''(a)=-6x^{-4} \qquad f'''(-1)=-6$$

$$f''''(a)=24x^{-5} \qquad f''''(-1)=-24$$

So:

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{n}(a)}{n!}(x-a)^{n}=-1-1(x+1)-1(x+2)^{2}-1(x+3)^{3}+\ldots$$

Or:

$$f(x)=\sum_{n=0}^{\infty}-1(x+1)^{n}$$