ARITHMETIC SEQUENCES AND SERIES

An arithmetic sequence is a sequence where each consecutive term is derived from the previous term by addition or subtraction of a fixed number called a common difference. The pattern of such a sequence is that we are always adding a fixed number to the previous term to get to the next term.

Every sequence that has a pattern in addition isn't necessarily an arithmetic sequence; it only is if you are always adding the SAME number each time.

The general term of any arithmetic sequence is

$$a_n = a_1 + (n-1)d$$

where a_1 is the first term of the sequence and d is the common difference.

An arithmetic series is simply the sum of the first n terms of an arithmetic sequence, given by the general formula:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

where a_1 is the first term of the sequence and a_n is the *n*th term of the sequence in question.

Simple math can be used to determine the quantity of any series.

Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

Determine the value of the following summation:

$$\sum_{n=15}^{47} 2n-5$$

Solution:

The first term of the sequence yields:

$$a_1 = 2(1)-5 = -3$$

The second, third, and fourth term are:

$$a_2 = 2(2)-5 = -1$$
$$a_3 = 2(3)-5 = 1$$
$$a_4 = 2(4)+5 = 3$$

From observation, we can see that each term will be two units larger than the previous, so this is an arithmetical sequence and d=2. Note that this summation starts at n=15, not at n=1.

The quickest way to determine the value of this series is to find the sum of all 47 terms minus the first 14 terms, which will give us a result for the sum from term 15 through 47.

Recall that an arithmetic series is simply the sum of the first n terms of an arithmetic sequence, given by the general formula:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Where $a_1 = 7$, $a_{14} = 23$, and $a_{47} = 89$

Therefore:

$$S_{14} = \frac{14}{2}(-3+23) = 140$$

and

$$S_{47} = \frac{47}{2}(-3+89) = 2021$$

Subtracting these two values:

$$\sum_{n=15}^{47} 2n-5 = 2021-140 = 1881$$

Therefore:

$$\sum_{n=1}^{14} 2n-5 = 1881$$