ARITHMETIC SEQUENCES AND SERIES

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Find the n^{th} term and the first three terms of an arithmetic sequence with a 4th term equal to 93 and an 8th term equal to 65

Problem 2:

Find the n^{th} and the 26th term of an arithmetic sequence where:

$a_5 = 5$ and $a_{12} = 159$

Problem 3:

Find the value of *n* for which the following summation holds true:

$$\sum_{i=1}^{n} 0.25i + 2 = 21$$

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Solution 1:

Since a_4 and a_8 are four terms apart and knowing that this is an arithmetic sequence, we can conclude that:

$$a_4 = a_8 + 4d$$
$$65 = 93 + 4d$$

The common difference d is then:

$$d = -7$$

The general n^{th} term of any arithmetic sequence is:

$$a_n = a_1 + (n-1)d$$

Using the known 4th term:

$$a_4 = a_1 + (4-1)(-7) = 65$$

So:

$$a_1 = 114$$

Now that we have:

$a_1 = 114$ and $d = -7$

We can conclude that the n^{th} term of this sequence can be defined as:

$$a_n = 114 - 7(n-1)$$

And the first three terms are:

$$a_1 = 114 - 7(1-1) = 114$$
$$a_2 = 114 - 7(2-1) = 107$$
$$a_3 = 114 - 7(3-1) = 100$$

Solution 2:

These two terms are 12 – 5 = 7 places apart, so, from the definition of an arithmetic sequence:

$$a_{12} = a_5 + 7d$$

$$159 = 5 + 7d$$

$$d = 22$$

The general n^{th} term of any arithmetic sequence is:

$$a_n = a_1 + (n-1)d$$

Using the known 4th term:

$$a_5 = a_1 + (5-1)(22) = 5$$

So:

$$a_1 = -83$$

Now that we have:

$a_1 = -83$ and d=22

We can conclude that the n^{th} term of this sequence can be defined as:

$$a_n = -83 + (n-1)22$$

And the 26th term is:

$$a_{26} = -83 + (26-1)22 = 467$$

Solution 3:

The first four terms of the sequence are:

$$a_1 = .25(1) + 2 = 2.25$$

$$a_2 = .25(2) + 2 = 2.50$$

$$a_3 = .25(3) + 2 = 2.75$$

$$a_4 = .25(4) + 2 = 3.00$$

From routine observation we note that the common difference *d* is:

$$d = .25$$

Therefore, we know that the general formula for a arithmetic series is:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Where

$$s_n = 21$$
$$a_1 = 2.25$$
$$a_n = .25n + 2$$

Plugging these values in to the equation and solving for n:

$$21 = \frac{n}{2}(2.25 + (.25n + 2))$$
$$42 = n(2.25 + .25n + 2)$$

$$.25n^2 + 4.25n - 42 = 0$$

Solving the quadratic we find that n=-24 and n=7. We know that a negative number can't be the number, so conclude that n=7.