DOT PRODUCT

The Dot Product is one of two operations derived from the multiplication of two vectors.

Given two vectors $\vec{a} = < a_1, a_2, a_3 >$ and $\vec{b} = < b_1, b_2, b_3 >$, the dot product is found using the following formula:

$$\vec{a} \bullet \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

The difference between the dot product and the cross product is that the result of a dot product is a single value, which is why it is sometimes referred to as the scalar product.

There are certain properties and relationships that we must note regarding the Dot Product, they are as follows:

$$c\vec{A} \bullet \vec{B} = c(\vec{A} \bullet \vec{B})$$
$$(\vec{A} + \vec{B}) \bullet \vec{C} = \vec{A}\vec{C} \bullet \vec{B}\vec{C}$$
$$\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}$$

Given two vectors $\vec{a} = < a_1, a_2, a_3 >$ and $\vec{b} = < b_1, b_2, b_3 >$ with an angle Θ between them, where Θ is between 0 and π, the dot product can be found using the following formula:

$$\left|\vec{a} \bullet \vec{b}\right| = \left|\vec{a}\right|\left|\vec{b}\right| \cos\Theta$$

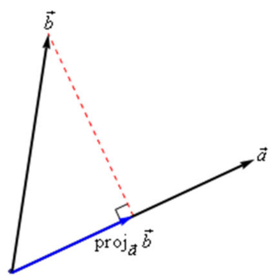
This formula is typically used with the traditional dot product formula to determine the angle between two vectors.

There are a number of general applications that knowledge of the dot product will need to exist to complete, one being the application of Projections.

Given two vectors $\vec{a}$ and $\vec{b}$ we want to determine the projection of $\vec{b}$ on to $\vec{a}$ denoted as $proj_{\vec{a}}\vec{b}$ is given by the formula:

$$proj_{\vec{a}}\vec{b} = \frac{\vec{a} \bullet \vec{b}}{\left|\vec{a}\right|^2}\vec{a}$$

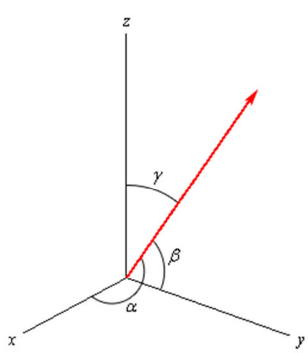
The result of a projection is a new vector as illustrated:



It is important to note that the projection of $\vec{a}$ on to $\vec{b}$ will not be the same. The formula is generally the same but is written as:

$$proj_{\vec{b}}\vec{a} = \frac{\vec{a} \bullet \vec{b}}{\left|\vec{b}\right|^2}\vec{b}$$

Lastly, the dot product can be used to determine the directional cosines of a vector. Say you are given the vector $\vec{a} = < a_1, a_2, a_3 >$ originating at the origin, illustrated as:



This vector forms certain angles with the x, y, a z axes. These angles are known as directional cosines and are determined using the following formulas:

$$\cos\alpha = \frac{a_1}{|\vec{a}|}$$

$$\cos\beta = \frac{a_2}{|\vec{a}|}$$

$$\cos\gamma = \frac{a_3}{|\vec{a}|}$$

Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

Given $\vec{a} = \vec{i}$ and $\vec{b} = 2\vec{i} + 2\vec{j}$ determine the dot product $\vec{a} \times \vec{b}$

Solution:

The Dot Product is derived from the multiplication of two vectors.

Given two vectors $\vec{a} =< a_1, a_2, a_3 >$ and $\vec{b} =< b_1, b_2, b_3 >$, the dot product is found using the following formula:

$$\vec{a} \bullet \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

In this problem, $\vec{a} =< 1,0,0 >$ and $\vec{b} =< 2,2,0 >$, therefore the dot product is:

$$\vec{a} \bullet \vec{b} = 1(2) + 0(2) + 0(0) = 2$$