ELEMENTARY ROW OPERATIONS

Elementary matrix operations can be of use in many matrix algebra applications, most notably in finding the inverse of a matrix and solving simultaneous linear equations.

There are three kinds of elementary matrix operations.

1. Interchanging of two rows.
2. Multiplying each element in a row by a non-zero number.
3. Multiplying a row by a non-zero number and add the result to another row.

These operations can be performed the same on columns as they are on rows. When they are performed on rows they are called elementary row operations. Conversely, when performed on columns they are called elementary column operations.

It is important to know what an identity matrix is as well as the notation that each represents elementary row operation.

Identity Matrix:_ A identity matrix is a nxn matrix with 1's in the diagonal and 0's elsewhere, as an example a 2x2 identity matrix takes the form:

$$I = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

Notation:

- Interchanging rows *i* and *j* is written as R_i <--> R_j
- Multiplying row *i* by *s* is written as sR_i --> R_i
- Add *s* times row *i* to row *j* is written as $sR_i + R_j$ --> R_j

Again, column notation follows the same format as elementary row operation notation.

To perform an elementary row operation on an *r* x *c* matrix, the following steps are completed:

1. Find E, the elementary row operator, by applying the desired operation to an *r* x *r* identity matrix.
2. Carry out the elementary row operation by premultiplying A by E.

To perform an elementary column operation on an *r* x *c* matrix, the following steps are completed:

1. Find E, the elementary column operator, by applying the operation to an *c* x *c* identity matrix.
2. Carry out the elementary column operation by postmultiplying A by E.

It's important to note that when completing elementary row operations, the elementary row operator is created from an identity matrix having the same number of rows and columns as there are rows in the original matrix. On the other hand, when completing elementary column operations the elementary column operator is created from an identity matrix having the same number of rows and columns as there are columns in the original matrix. Just as important, in row operations, you are premultiplying the original matrix, whereas in column operations you are postmultiplying the original matrix.

Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

Complete the row operation $R2-2R1$ on the following matrix:

$$A=\begin{bmatrix}4 & 2\\3 & 3\end{bmatrix}$$

Solution:

To perform an elementary row operation on an *r* x *c* matrix, the following steps are completed:

1. Find E, the elementary row operator, by applying the desired operation to an *r* x *r* identity matrix.

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad R2 - 2R1 \Rightarrow E = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

2. Carry out the elementary row operation by premultiplying A by E.

$$EA = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1(4) + 0(3) & 1(2) + 0(3) \\ -2(4) + 1(3) & -2(2) + 1(3) \end{bmatrix}$$

Therefore, the row operation $R2 - 2R1$ on $A = \begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$ is:

$$EA = \begin{bmatrix} 4 & 2 \\ -5 & -1 \end{bmatrix}$$