MATRIX MULTIPLICATION

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Multiply the following matrices:

$$\begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 2 \\ 4 & 11 & 2 \end{bmatrix}$$

Problem 2:

Determine the resulting matrix from:

$$\begin{bmatrix} \cos(60) & -\sin(60) & 0 \\ \sin(60) & \cos(60) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$$

Problem 3:

Given:

$$A = \begin{bmatrix} -2 & 1 & 7 \\ 3 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 4 & -1 & 5 \end{bmatrix}$$

Determine AB and BA

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Solution 1:

Recall that when given two matrices A and B:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \quad B=\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}$$

The multiplication of A and B is:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}=\begin{bmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} \end{bmatrix}$$

Therefore:

$$\begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 2 \\ 4 & 11 & 2 \end{bmatrix}=\begin{bmatrix} 3(0)+-1(4) & 3(-1)+-1(11) & 3(2)+-1(2) \\ 1(0)+2(4) & 1(-1)+2(11) & 1(2)+2(2) \\ 6(0)+1(4) & 6(-1)+1(11) & 6(2)+1(2) \end{bmatrix}$$

So:

$$\begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 2 \\ 4 & 11 & 2 \end{bmatrix}=$$

Solution 2:

Convert the trigonometric expressions such that:

$$\begin{bmatrix} \cos(60) & -sin(60) & 0 \\ sin(60) & cos(60) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} .5 & -.866 & 0 \\ .866 & .5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore, carrying out the multiplication of the two matrices gives:

$$\begin{bmatrix} .5 & -.866 & 0 \\ .866 & .5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} .5(2)+4(-.866)+0(0) \\ .866(2)+.5(4)+0(0) \\ 0(2)+0(4)+1(0) \end{bmatrix}$$

So:

$$\begin{bmatrix} \cos(60) & -sin(60) & 0 \\ sin(60) & cos(60) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} -2.46 \\ 3.73 \\ 0 \end{bmatrix}$$

Solution 3:

With the matrices:

$$A = \begin{bmatrix} -2 & 1 & 7 \\ 3 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix} \quad \text{and} \quad B =$$

Multiplying AB is not possible because the number of columns in the first does not match the number of rows in the second.

However, multiplying BA is possible because the number of columns in the first match the number of rows in the second.

Multiplying BA gives:

$$\begin{bmatrix}4 & -1 & 5\end{bmatrix}\begin{bmatrix}-2 & 1 & 7\\ 3 & -1 & 0\\ 0 & 2 & -1\end{bmatrix}=\begin{bmatrix}-8-3+0 & 4+1+10 & 28+0-5\end{bmatrix}$$

So:

$$\begin{bmatrix}4 & -1 & 5\end{bmatrix}\begin{bmatrix}-2 & 1 & 7\\ 3 & -1 & 0\\ 0 & 2 & -1\end{bmatrix}=\begin{bmatrix}-11 & 15 & 23\end{bmatrix}$$