RULES OF LOGARITHMS

Many natural phenomenon vary as a power function a^x, so that they display exponential change, or growth, over widely measurable ranges of the exponent x. We as engineers are therefore interested in the value of this exponent, which is called a logarithm.

Often when we are analyzing power functions, $y = a^x$, it is the y that is the known value, and the exponent x is the value that we wish to determine.

So in order to do this, we must rearrange the equation so that it is in the form of $x = f(y)$, which turns out to be the inverse of a power function, and is called a logarithm. Written as:

$$x = \log_a y$$

Where y is called the argument of the logarithm.

When encountering logarithms, there are many rules are used to simplify the process of solving for the required information.

When the argument of a logarithm is a product of two numbers, $\log_a(y \cdot z)$, it can be rewritten as the sum of the two logarithms, such as:

$$\log_a(y \cdot z) = \log_a(y) + \log_a(z)$$

When an argument of a logarithm is a quotient of two numbers, $\log_a(\frac{y}{z})$, it can be rewritten as the difference of the two logarithms, such as:

$$\log_a(\frac{y}{z}) = \log_a(y) - \log_a(z)$$

When an argument of a logarithm is a power of two numbers, $\log_a(b^y)$, it can be rewritten as the difference of the a multiple of logarithm, such as:

$$\log_a(b^y) = y\log_a(b)$$

Because any number raised to the power of 1 is equal to itself, $a^1 = a$, the following logarithmic identity always holds true:

$$\log_a(a) = 1$$

On the hand, because any number raised to the power of 0 is equal to 1, the following logarithmic identity always holds true:

$$\log_a(1) = 0$$

In all mathematical studies, the base 10 logarithm is very important and is called the "common logarithm" and typically the subscript after the log is dropped:

$$x = \log_{10}(y) = \log(y)$$

So if you see log by itself, it is naturally a base 10 problem.

Another base that shows up naturally in math is the transcendental number *e*, which is equal to 2.7182... As noted in the power functions review, the power of this base is equal to its own derivative.

When e is the base of a logarithm, it is known as the natural logarithm, typically abbreviate as ln:

$$x = \log_e(y) = \ln y$$

Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

Condense the following expression:

$$4 \cdot \ln 2 + 2 \cdot \ln x - \ln y$$

Solution:

When asked to condense the logarithm, the goal is to write the formula as a signle logarithm with a coefficient of 1.

The first step is to move the coefficients from the front in to the exponents such that:

$$4 \cdot \ln 2 + 2 \cdot \ln x - \ln y = \ln 2^4 + \ln x^2 - \ln y$$

Now condense the addition and subtractions, into multiplication and division, such that:

$$\ln 2^4 + \ln x^2 - \ln y = \ln 2^4 x^2 - \ln y = \ln \frac{2^4 x^2}{y}$$

So, the condensed form would be:

$$4 \cdot \ln 2 + 2 \cdot \ln x - \ln y = \ln \frac{16x^2}{y}$$