RULES OF POWER FUNCTIONS

Many natural phenomenon vary as a power function a^x, so that they display exponential change, or growth, over widely measurable ranges of the exponent x.

When working with power functions, there are many rules that can be used to simplify the process of solving.

The first rule is the power of products. When we are given a power function where the base is made up of a product of two numbers, say ab, and is raised to the x, that function can be rewritten as:

$$(ab)^x = a^x b^x$$

If a function is a product of two power functions of the same base being raised to different powers, it can be rewritten by adding the exponents and setting them together with the common base:

$$a^w a^x = a^{w+x}$$

If a function is a quotient of two power functions of the same base being raised to different powers, it can be rewritten by subtracting the exponents and setting them together with the common base:

$$\frac{a^w}{a^x} = a^{w-x}$$

If the base of a power function is a power function itself, it can be rewritten by multiplying the exponents together and setting it against the original base:

$$(a^w)^x = a^{wx}$$

A couple of common, yet important special cases of power functions are those when the exponents are equal to 0 and 1.

When any base is raised to the power of 1, the base is being multiplied by itself once so that:

$$a^1 = a$$

When any base is raised to the power of 0 the result is 1, so that:

$$a^0 = 1$$

It was determined that a power function being divided by another power function of the same base is a new power function of the common base raised to the subtraction of the exponents. When the denominator of these power functions has a larger exponent than that in the numerator, we will get a negative exponent, so with that, a base raised to a negative exponent can be rewritten as:

$$a^{-x} = \frac{1}{a^x}$$

When a base is raised by a fraction, $\frac{m}{n}$, it can be rewritten as:

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

One particular base of singular importance in Math is that of the transcendental number, or e, which is:

$$e = 2.7182\ldots$$

The base e shows up naturally in calculus because the power of this base is equal to its own derivative:

$$y = e^x$$
$$\frac{dy}{dx} = e^x$$

The base power e power function, e^x, is called the exponential function and is sometimes abbreviated as exp(x).

Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

Condense the following expression:

$$5^2\left(\frac{5^7}{5^3}\right)$$

Solution:

When asked to condense a power function, the goal is to write the formula as a single power function with a common base.

The first step is to condense the quotient knowing that if a function is a quotient of two power functions of the same base being raised to different powers, it can be rewritten by subtracting the exponents and setting them together with the common base:

$$5^2\left(5^{7-3}\right) \Rightarrow 5^2\left(5^4\right)$$

Now condense using the power of products:

$$5^2\left(5^4\right)=5^6$$

So, the condensed form would be:

$$5^2\left(\frac{5^7}{5^3}\right)=5^6$$