HIGHER ORDER HOMOGENOUS DIFFERENTIAL EQUATIONS

When given an equation in the general form:

$$y'' + p(t)y' + q(t)y = 0$$

Use the following process to determine a general solution in the form:

$$y(t) = c_1 y_1(t) + c_2 y_2(t)$$

The first step is to put the equation in to its characteristic form simply by adding an r variable in the place of the y variables, so that:

$$ar^2 + br + c = 0$$

Next, determine the roots of the equation by either factoring or using the quadratic formula. The values of r will then determine the form of $y_1(t)$ and $y_2(t)$ using the following table:

Roots	General Solution
Real and distinct roots: r_1, r_2; $b^2 - 4ac > 0$	$y(t) = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
Real and equal roots: $r_1 = r_2$; $b^2 - 4ac = 0$	$y(t) = c_1 e^{rx} + c_2 x e^{rx}$
Complex roots: $r_1, r_2 = \alpha \pm j\omega$; $b^2 - 4ac < 0$	$y(t) = e^{\alpha x}(c_1 \cos \omega x + c_2 \sin \omega x)$

Use the initial conditions, if given, to determine the unknown coefficients.

Concept Example:

CONCEPT INTRODUCTION

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

Solve the following differential equation:

$$y'' + 60y' + 500y = 0$$

Solution:

The first step is to put the equation in to its characteristic form simply by adding an r variable in the place of the y variables. The characteristic equation is:

$r^2 + 60r + 500 = 0$ which factors out to $(r+50)(r+10) = 0$

The roots are r=-50 and r=-10, which shows that we have two real distinct roots, making the general solution:

$$y_h = c_1 e^{-50t} + c_2 e^{-10t}$$

If initial conditions were given we could determine the unknown coefficients from here.