INTEGRATION BY SUBSTITUTION

Many integrals can be computed by means of a change of variables, more commonly called u-substitution.

This substitution method amounts to applying the Chain Rule in reverse and is define through illustrating generally as:

Given an integral:

$$\int f(g(x))g'(x)dx$$

Let $u = g(x)$ and $du = g'(x)dx$ and substitute these terms in to the integral so that:

$$\int f(u)du$$

Then:

$$\int f(g(x))g'(x)dx = \int f(u)du = F(u) = F(g(x)) \text{ where F is the antiderivitive of f}$$

This result can be checked by differentiating it using the Chain Rule to see if the original integral is obtained.

Definite integrals are solved using one of two methods

1. Plug back in the original substitution and solve using the original limits of integration
2. Maintain the substituted values and change the limits of integration to terms of the u-substitution.

So the process is as followed:

Given an Indefinite Integral:

1. First identify that you have a function of a function. This skill comes with practice to identify candidates.
2. Identify u and then find du that is appropriate for the expression.
3. Integrate using normal techniques.
4. Substitute back the values for u for indefinite integrals.
5. Don't forget the constant of integration for indefinite integrals.

Given a Definite Integral:

1. First identify that you have a function of a function. This skill comes with practice to identify candidates.
2. Identify u and then find du that is appropriate for the expression.
3. Change limits for definite integrals (If you choose this route)
4. Integrate using normal techniques.

Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

Evaluate the integral:

$$\int (x-3)^{10}\,dx$$

Solution:

Observing the general integral:

$$\int f(g(x))g'(x)dx$$

Let $u = x - 3$ and $du = 1dx$ and substitute these terms in to the integral so that:

$$\int (u)^{10}\,du$$

Then:

$$\int (x-3)^{10}\,dx = \int (u)^{10}\,du = \frac{u^{11}}{11} + c$$

Substituting our value back in to the equation we get:

$$\int (x-3)^{10}\,dx = \frac{(x-3)^{11}}{11} + c$$

This result can be checked by differentiating it using the Chain Rule to see if the original integral is obtained.

$$\left(\frac{(x-3)^{11}}{11} + c\right)' = (x-3)^{10}$$