INTEGRATION BY SUBSTITUTION

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Evaluate the integral:

$$\int \frac{x}{\sqrt{9+x^2}}dx$$

Problem 2:

Evaluate the definite integral:

$$\int_0^2 x\cos(x^2+1)dx$$

Problem 3:

Evaluate the integral:

$$\int e^{5x+2}dx$$

INTEGRATION BY SUBSTITUTION

Solution 1:

This integral can be rewritten as:

$$\int x(9+x^2)^{-\frac{1}{2}}dx$$

Observing the general integral:

$$\int f(g(x))g'(x)dx$$

Let $u=9+x^2$ and $\frac{du}{dx}=2x$. Since du doesn't equal x, we will need to arrange the formula for du to ensure we maintain the integrity of the original integral.

$$\frac{du}{dx}=2x \Rightarrow \frac{du}{2}=xdx$$

Now substitute these terms in to the integral so that:

$$\int \frac{1}{2}(u)^{-\frac{1}{2}}du \Rightarrow \frac{1}{2}\frac{(u)^{\frac{1}{2}}}{\frac{1}{2}}+c$$

Or:

$$(u)^{\frac{1}{2}}+c$$

Substituting our value back in to the equation we get:

$$\int x(9+x^2)^{-\frac{1}{2}}dx=(9+x^2)^{\frac{1}{2}}+c$$

Solution 2:

Observing the general integral:

$$\int f(g(x))g'(x)dx$$

Let $u = x^2 + 1$ and $\frac{du}{dx} = 2x$. Since du doesn't equal x, we will need to arrange the formula for du to ensure we maintain the integrity of the original integral.

$$\frac{du}{dx} = 2x \Rightarrow \frac{du}{2} = xdx$$

Recall that definite integrals can be solved using one of two methods:

1. Plug back in the original substitution at the end and solve using the original limits of integration

2. Maintain the substituted values at the end and change the limits of integration to terms of the u-substitution.

In this problem, the limits of integration will be converted with respect to u. Note the value of the limits are originally:

$$\int_0^2 x\cos(x^2 + 1)dx$$

Using the substation, the limits convert to:

$$u = x^2 + 1 = 0^2 + 1 = 1$$
$$u = x^2 + 1 = 2^2 + 1 = 5$$

Now substitute all the substituted and converted terms back in to the integral so that:

$$\frac{1}{2}\int_1^5 \cos(u)du = \frac{1}{2}\left|\sin u\right|_1^5$$

The answer is then:

$$\int_0^2 x\cos(x^2+1)dx = \frac{1}{2}(\sin 5 - \sin 1)$$

Solution 3:

Observing the general integral:

$$\int f(g(x))g'(x)dx$$

Let $u = 5x+1$ and $\frac{du}{dx} = 5$. Since du doesn't equal x, we will need to arrange the formula for du to ensure we maintain the integrity of the original integral.

$$\frac{du}{dx} = 5 \Rightarrow \frac{1}{5}du = dx$$

Now substitute these terms in to the integral so that:

$$\int \frac{1}{5}e^u du$$

Or:

$$\frac{1}{5}e^u + c$$

Substituting our value back in to the equation we get:

$$\int e^{5x+2}dx = \frac{1}{5}e^{5x+1} + c$$