INTEGRATION BY PARTS

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Integrate $\int x \cos 3x \, dx$

Problem 2:

Integrate $\int \frac{In\, x}{x^5} \, dx$

Problem 3:

Integrate $\int arcsin\, 3x \, dx$

INTEGRATION BY PARTS

Solution 1:

Integrate using the four steps of integration by substitution:

1. Write the given integral:

$$\int x\cos 3x\, dx$$

2. Define the intermediary functions u and v and there derivatives such that:

$$u = x \qquad dv = cos3x\, dx$$

$$du = dx \qquad v = \left(\frac{1}{3}\right) sin3x\,.$$

3. Plug the values in to the general formula:

$$\int udv = uv - \int vdu$$

$$\int x\cos 3xdx = \frac{1}{3}x\sin 3x - \int \frac{1}{3}\sin 3xdx$$

4. Complete the new integral on the right side of the equation:

$$\frac{1}{3}\int \sin 3x\, dx = -\frac{1}{9}cos3x + c$$

Therefore, $\int x\cos 3x\, dx = \frac{1}{3}xsin3x + \frac{1}{9}cos3x + C$

Solution 2:

Integrate using the four steps of integration by substitution:

1. Write the given integral:

$$\int \frac{In\, x}{x^5} dx$$

2. Define the intermediary functions u and v and there derivatives such that:

$$u = In\, x \qquad dv = \frac{1}{x^5} dx = x^{-5}\, dx$$

$$du = \frac{1}{x} dx \qquad v = \frac{x^{-4}}{-4} = \frac{-1}{4x^4}.$$

3. Plug the values in to the general formula:

$$\int u dv = uv - \int v du$$

$$\int \frac{In\, x}{x^5} dx = \left(In\, x\right)\left(\frac{-1}{4x^4}\right) - \int \left(\frac{-1}{4x^4}\right)\left(\frac{1}{x}\right) dx$$

$$= -\frac{In\, x}{4x^4} + \left(\frac{1}{4}\right)\int \frac{1}{x^5} dx$$

4. Complete the new integral on the right side of the equation:

$$\left(\frac{1}{4}\right)\int x^{-5}\, dx = \left(\frac{1}{4}\right)\frac{x^{-4}}{-4} + c$$

Therefore, $\int \frac{In\, x}{x^5} dx = -\frac{In\, x}{4x^4} - \frac{1}{16x^4} + c$

Solution 3:

Integrate using the four steps of integration by substitution:

1. Write the given integral:

$$\int arcsin\ 3x\ dx$$

2. Define the intermediary functions u and v and there derivatives such that:

$$u = \arcsin 3x \qquad dv = dx = (1)dx$$

$$dv = \frac{3}{\sqrt{1-9x^2}}dx \qquad v = x$$

3. Plug the values in to the general formula:

$$\int udv = uv - \int vdu$$

$$\int \arcsin 3x\, dx = x \arcsin 3x - \int x \frac{3}{\sqrt{1-9x^2}}dx$$

$$= x \arcsin 3x - 3\int \frac{x}{\sqrt{1-9x^2}}dx.$$

4. Complete the new integral on the right side of the equation:

Let's use u-substitution to simplify the integration of the new integral:

Let

$u = 1-9x^2$ and $du = -18x\,dx$ or $\left(-\frac{1}{18}\right)du = x\,dx$

Then:

$$3\int \frac{x}{\sqrt{1-9x^2}}dx = 3\int \frac{1}{\sqrt{1-9x^2}}xdx$$

Using the substitutions:

$$3\int \frac{1}{\sqrt{1-9x^2}}xdx = 3\int \frac{1}{\sqrt{u}}\left(-\frac{1}{18}\right)du$$

$$-\frac{1}{6}\int u^{-\frac{1}{2}}du = -\frac{1}{6}\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = -\frac{1}{3}(1-9x^2)^{\frac{1}{2}} + c$$

Therefore, $\int arcsin\,3x\,dx = x\arcsin 3x + \left(\frac{1}{3}\right)\sqrt{1-9x^2} + C$