CURVATURE

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Determine the radius of curvature at (1, -1) on the curve $y = x^2 - 3x + 1$

Problem 2:

Find the radius of curvature at any point on the curve $f(x) = alog(\sec\left(\frac{x}{a}\right))$

Problem 3:

Find the radius of curvature of the function $f(x) = 2\log\sin\left(\frac{x}{2}\right) at\ x = \frac{\pi}{3}$

CURVATURE

Solution 1:

To find the radius of curvature, we first need to determine the derivative of f(x) which is:

$$f'(x) = 2x - 3$$

And the Second Derivative is:

$$f''(x) = 2$$

Evaluate each of the derivatives at the point (1,-1)

$$f'(1) = 2(1) - 3 = -1$$
$$f''(1) = 2$$

Now substitute these values in to the formula to determine the radius of curvature at at any point x:

$$\rho = \frac{\left(1 + [f'(x)]^2\right)^{\frac{3}{2}}}{|f''(x)|}$$

$$\rho = \frac{\left(1 + [-1]^2\right)^{\frac{3}{2}}}{|2|} = \sqrt{2}$$

Solution 2:

To find the radius of curvature, we first need to determine the derivative of f(x) which is:

$$f'(x) = a \times \frac{1}{\sec(\frac{x}{a})} \cdot \sec(\frac{x}{a})\tan(\frac{x}{a}) \cdot \frac{1}{a} = \tan(\frac{x}{a})$$

And the Second Derivative is:

$$f''(x) = \sec^2(\frac{x}{a}) \cdot \frac{1}{a}$$

Substituting these values in to the formula to determine the radius of curvature at at any point x:

$$\rho = \frac{\left(1 + [f'(x)]^2\right)^{\frac{3}{2}}}{|f''(x)|}$$

$$\rho = \frac{\left(1 + \tan^2(\frac{x}{a})\right)^{\frac{3}{2}}}{\left|\sec^2(\frac{x}{a}) \cdot \frac{1}{a}\right|}$$

Which can be simplified to:

$$\rho = a\sec(\frac{x}{a})$$

Solution 3:

To find the radius of curvature, we first need to determine the derivative of f(x) which is:

$$f'(x) = 2 \times \frac{1}{\sin\left(\frac{x}{2}\right)} \times cos\left(\frac{x}{2}\right) \times \frac{1}{2} = cot(\frac{x}{2})$$

And the Second Derivative is:

$$f^{''}(x) = -\csc^2\left(\frac{x}{2}\right) \times \frac{1}{2}$$

Evaluate each of the derivatives at $\frac{\pi}{3}$

$$f^{'}\left(\frac{\pi}{3}\right) = cot\left(\frac{\pi}{6}\right) = \sqrt{3}$$

$$f^{''}\left(\frac{\pi}{3}\right) = -\frac{1}{2}\csc^2\left(\frac{\pi}{6}\right) = -2$$

Now substitute these values in to the formula to determine the radius of curvature at at $\frac{\pi}{3}$:

$$\rho = \frac{\left(1 + \left[f'(x)\right]^2\right)^{\frac{3}{2}}}{\left|f''(x)\right|}$$

$$\rho = \frac{\left(1 + \left[\sqrt{3}\right]^2\right)^{\frac{3}{2}}}{\left|-2\right|} = -4$$