PARTIAL DERIVATIVES

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Determine the partial derivative with respect to x of:

$$f(x,y) = 2x\ \sin(x^2 y)$$

Problem 2:

Find all the partial derivatives for the function:

$$f(x,y,z) = x\cos z + x^2 y^3 e^z$$

Problem 3:

Determine the partial derivatives of:

$$f(x,y) = x^2 \sin y + y^2 \cos x$$

PARTIAL DERIVATIVES

Solution 1:

To find the partial derivative of $f(x, y)$ with respect to x, we need to treat all the *y*'s as constants and then differentiate the *x*'s.

In this case, we have a function multiplied by a composite function, so we are going to need to use a combination of the product rule and the Chain Rule.

Recall these rules:

Product Rule: If f and g are differentiable functions, then:

$$[f(x)g(x)]' = f(x)g'(x) + g(x)f'(x)$$

A phrase to remember the Product Rule is: First times the derivative of the second, plus the second times the derivative of the first

Chain Rule: If a composite function made up of two other functions, where $g(x)$ is substituted for x into a function $f(x)$ such as:

$$(f \circ g)(x) = f(g(x))$$

The chain rule states that if f and g are differentiable functions and $F(x) = f(g(x))$, then F is differentiable and the derivative of F is given by:

$$F'(x) = f'(g(x))g'(x)$$

So we are going to define each function and its derivatives and then plug them in as we go to define the partial derivative with respect to x.

So:

$f(x) = 2x$ and $f^{'}(x) = 2$

$g(x) = \sin(x^2 y)$ and $g^{'}(x) = \cos(x^2 y) \cdot 2xy$

Plugging in the values we find that:

$$\frac{\partial f}{\partial x} = 2x \cdot \cos\left(x^2 y\right) \cdot 2xy + \sin\left(x^2 y\right) \cdot 2 = 4x^2 y cos\left(x^2 y\right) + 2\sin(x^2 y)$$

Solution 2:

To find the partial derivative of $f(x,y,z)$ with respect to x, we need to treat all the *y*'s and z's as constants and then differentiate the *x*'s, so:

$$\frac{\partial f}{\partial x} = \cos z + 2xy^3 e^z$$

To find the partial derivative of $f(x,y,z)$ with respect to y, we need to treat all the *x*'s and z's as constants and then differentiate the *y*'s, so:

$$\frac{\partial f}{\partial y} = 3x^2 y^2 e^z$$

To find the partial derivative of $f(x,y,z)$ with respect to z, we need to treat all the *x*'s and y's as constants and then differentiate the *z*'s, so:

$$\frac{\partial f}{\partial z} = -x\sin z + x^2 y^3 e^z$$

Solution 3:

To find the partial derivative of $f(x,y)$ with respect to x, we need to treat all the *y*'s as constants and then differentiate the *x*'s, so:

$$f_x = 2x\sin y - y^2 \sin x$$

To find the partial derivative of $f(x,y)$ with respect to y, we need to treat all the *x*'s as constants and then differentiate the *y*'s, so:

$$f_y = x^2 \cos y + 2y\cos x$$