CRITICAL POINTS

Given any function, $f(x)$, it is said that at x=c there is a critical point if either of the following scenarios hold true:

$f'(c)=0$
$f'(c)$ doesn't exist

Finding critical points for a polynomial function is fairly simple, granted the degree doesn't get overly large making it difficult to determine the roots. However, often times we encounter functions that are more difficult and will require factoring and other means to determine where the function has a critical point.

Polynomials are fairly simple functions to find critical points for provided the degree doesn't get so large that we have trouble finding the roots of the derivative.

When a critical point is defined, the Second Derivative Test can be used to determine if the point is a maxima or minima of the particular function. The Second Derivative states that:

- If $f''(c)>0$, then $f(x)$ is increasing in the interval around *c*. Since $f'(c)=0$, then $f(x)$ must be negative to the left of *c* and positive to the right of *c*. Therefore, *c* is a local minimum.

- If $f''(c)<0$, then $f(x)$ is decreasing in the interval around *c*. Since $f'(c)=0$, then $f(x)$ must be positive to the left of *c* and negative to the right of *c*. Therefore, *c* is a local maximum.

Concept Example:

CONCEPT INTRODUCTION

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

Find the critical points of the function:

$$f(x) = \frac{x}{1+x^2}$$

Solution:

The first step is to determine the critical points. The function *f*(*x*) is rational and is defined for any *x*.

So, given any function, $f(x)$, it is said that at x=c there is a critical point if either of the following scenarios hold true:

$f'(c) = 0$
$f'(c)$ doesn't exist

Using the Quotient Rule states, which states:

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

The first derivative of $f(x)$ is:

$$f'(x) = \frac{\left(1+x^2\right) - 2x^2}{\left(1+x^2\right)^2} = \frac{1-x^2}{\left(1+x^2\right)^2}$$

$f'(x) = 0$ implies that critical points will be located as $x = \pm 1$.