DERIVATIVES

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Differentiate $f(x) = (x^3 + 2x)(5x^2 + 7x + 9)$ using the product rule

Problem 2:

Differentiate $\mathrm{f}(\mathrm{x}) = \sqrt{2\mathrm{x}^2 - 8}$ using the chain rule

Problem 3:

Find the first, second, and third derivatives of $f(x) = 3x^4 + 2x^2 + 7x$

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Solution 1:

Recall that the Product Rule states that If h and g are differentiable functions, then:

$$[h(x)g(x)]' = h(x)g'(x) + g(x)h'(x)$$

Remember that the phrase to remember when dealing with the Product Rule is this: First times the derivative of the second, plus the second times the derivative of the first.

In this problem, let's define:

$$g(x) = x^3 + 2x$$

And

$$h(x) = 5x^2 + 7x + 9$$

Determining the derivatives of each of these functions:

$$g'(x) = 3x^2 + 2$$

And

$$h'(x) = 10x + 7$$

Applying the product Rule:

$$[h(x)g(x)]' = h(x)g'(x) + g(x)h'(x)$$

Substitute the values:

$$f'(x) = (3x^2 + 2)(5x^2 + 7x + 9) + (10x + 7)(x^3 + 2x)$$

Expanding:

$$f'(x) = (15x^4 + 21x^3 + 27x^2 + 10x^2 + 14x + 18) + (10x^4 + 20x^2 + 7x^3 + 14x)$$

Combing like terms, the derivative is:

$$f'(x) = 25x^4 + 28x^3 + 57x^2 + 28x + 18$$

Solution 2:

Recall that if a composite function made up of two other functions, where $g(x)$ is substituted for x into a function $f(x)$ such as:

$$(f \circ g)(x) = f(g(x))$$

The chain rule states that if f and g are differentiable functions and $F(x) = f(g(x))$, then F is differentiable and the derivative of F is given by:

$$F'(x) = f'(g(x))g'(x)$$

In this problem, let's define:

$$f(u) = \sqrt{u}$$

And

$$g(x) = 2x^2 - 8$$

Determining the derivatives of each of these functions:

$$f^{'(u)} = \left(\frac{1}{2}\right) u^{-\frac{1}{2}} = \frac{1}{2\sqrt{u}}$$

And

$$g'(x) = 4x$$

We now substitute these equations into the formula for $F^{'}(x)$ such that:

$$F^{'}(x) = f^{'}\left(g(x)\right) g^{'}(x)$$

$$= \left(\frac{1}{2\sqrt{(2x^2 - 8)}}\right)(4x) = \frac{4x}{2\sqrt{2x^2 - 8}}$$

The derivative is then:

$$F'(x) = \frac{2x}{\sqrt{(2x^2 - 8)}}$$

It is important to note that it is not necessary to write the function as a composition of two functions and find the derivative of each separately. With practice, all of the steps can be done at once. However, until the process becomes automatic, it's helpful so we make sure we keep track of each step.

Solution 3:

Recall that if the function $f(x)$ has a derivative $f'(x)$ which is differentiable, the derivative of it can be taken as well. The new function, $f''(x)$, is called the second derivative of $f(x)$. This process can continue as long as the function is differentiable.

So to find the first, second, and third derivative of $f\left(x\right)=3x^{4}+2x^{2}+7x$, we follow the process:

$$f'(x)=12x^{3}+4x+7$$

If we differentiate the first derivative, we can find $f''(x)$:

$$f''(x)=36x^{2}+4$$

Differentiating once more, we can find $f'''(x)$:

$$f'''(x)=72x$$