

HYPOTHESIS TESTING

A statistical hypothesis is an assumption about some population parameter, which can be either true or false.

Ideally, to determine the validity of a statistical hypothesis, one would examine an entire population. However, this is obviously not possible in most cases, so in exchange, we typically examine a random sample from the population. If the sample data is inconsistent with the hypothesis, then the hypothesis is rejected.

There are two types of statistical hypotheses:

1. The null hypothesis, which is denoted by H_0
2. The alternative hypothesis, denoted by H_A

There are two types of errors that can result from a hypothesis test:

- A Type I error occurs when we reject a null hypothesis when it is true. The probability of committing a Type I error is called the significance level and is often denoted by α .
- A Type II error occurs when we fail to reject a null hypothesis that is false. The probability of committing a Type II error is called Beta, denoted by β .

The analysis plan includes decision rules for rejecting the null hypothesis. The strength of evidence in support of a null hypothesis is measured by the P-value. If the P-value is less than the significance level, we reject the null hypothesis.

The most common hypothesis test encountered is that of testing a mean.

There are four steps to completing a hypothesis test:

1. State the hypotheses
2. Formulate an analysis plan

3. Analyze sample data
4. Interpret results

The first step requires that a null and alternative hypotheses are defined. The hypotheses are stated in such a way that they are mutually exclusive. That is, if one is true, the other must be false.

There are three sets of possible hypotheses.

Set	Null Hypothesis	Alternative Hypothesis	Tails
1	$\mu = M$	$\mu \neq M$	2
2	$\mu \geq M$	$\mu < M$	1
3	$\mu \leq M$	$\mu > M$	1

Each set of hypotheses makes a statement about how the population mean μ is related to a specified value M .

The first set of hypotheses is considered a two-tailed test, since an extreme value on either side of the sampling distribution would call for rejection of the null hypothesis. The other two sets are one-tailed tests, since an extreme value on only one side of the sampling distribution would cause rejection of the null hypothesis.

The next step is to develop an analysis plan. The analysis plan describes how to use sample data to accept or reject the null hypothesis. It defines the following characteristics:

- The significance level, which is typically equal to 0.01, 0.05, or 0.10; but any value between 0 and 1 can be used.
- The testing method, which typically can be the one-sample t-test to determine whether the hypothesized mean differs significantly from the observed sample mean.

Once the analysis plan is done, use the sample data to conduct a one-sample t-test. This involves finding the standard error, degrees of freedom, test statistic, and the P-

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value associated with the test statistic. To complete this step, use the following process:

- Compute the standard error (SE) of the sampling distribution, which is done with the equation:

$$SE = \frac{s}{\sqrt{n}}$$

- Determine the degrees of freedom (DF), which is equal to the sample size minus one or:

$$DF = n - 1$$

- Define the test statistic, a t-score, defined as:

$$t = \frac{(\bar{x} - \mu)}{SE}$$

Where \bar{x} is the sample mean, μ is the hypothesized population mean in the null hypothesis, and SE is the standard error.

- Finally, determine the P-value using the t-distribution table. The p-value is the probability of observing a sample statistic as extreme as the test statistic.
- Interpret the results. If the sample findings are unlikely, given the null hypothesis, the null hypothesis is rejected. This is done by comparing the P-value to the significance level, and rejecting the null hypothesis when the P-value is less than the significance level.

Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

In an attempt to encourage students to stop driving to campus, a certain University claims that on average it takes people 30 minutes to find a parking space when they get to school. Students don't believe it takes this long, and actually set out to prove the University wrong on their stance. In a test, a student monitored the time it took to park the last 5 times they drove to campus and reported that the mean time was 20 minutes. Assuming that this time is normal and that $\sigma = 6$ minutes, perform a hypothesis test with a significance level of .10 to see if the students claim is correct.

Solution:

Recall that there are four steps to completing a hypothesis test:

1. State the hypotheses
2. Formulate an analysis plan
3. Analyze sample data
4. Interpret results

So let's run through the steps:

Step 1: State the hypotheses

With the given information, we can state the hypothesis as:

$$H_0 : \mu \geq 30$$

$$H_A : \mu < 30$$

Step 2: Formulate an analysis plan

- The significance level is defined as .10
- The testing method will be the one-sample t-test

Step 3: Analyze sample data

- Compute the standard error (SE) of the sampling distribution, which is done with the equation:

$$SE = \frac{s}{\sqrt{n}} = \frac{6}{\sqrt{5}} = 2.68$$

- Determine the degrees of freedom (DF), which is equal to the sample size minus one or:

$$DF = n - 1 = 5 - 1 = 4$$

- Define the test statistic, a t-score, defined as:

$$t = \frac{(\bar{x} - \mu)}{SE} = \frac{(20 - 30)}{2.68} = -3.73$$

Where \bar{x} is the sample mean, μ is the hypothesized population mean in the null hypothesis, and SE is the standard error.

- Finally, determine the P-value using the t-distribution table. The p-value is the probability of observing a sample statistic as extreme as the test statistic. This is a single tail problem where a t-score of -3.73 and 4 degrees of freedom corresponds to a p-value of approximately .01

Step 4: Interpret results

- Comparing the P-value to the significance level, we find that $.01 < .1$, which calls for the rejection of the null hypothesis in exchange for the alternative hypothesis that it takes less than 30 minutes to park.