HYPOTHESIS TESTING

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

A sample of 40 transactions at a local shop averages $137 with a standard deviation of $30.20. Test whether or not the mean sales at this shop are different than $150. Assume a significance level of .01.

Problem 2:

An editor at a publishing company claims that the mean time to write an Engineering review manual is at most 15 months. A sample of 16 engineering textbook authors are randomly selected and it's found that the mean time taken to write a review manual was 12.5 months with a standard deviation of 3.6 months. Using a significance level of 0.025, is the editor's claim accurate?

Problem 3:

At one point in time, it was determined that the mean salary for an entry level engineer in the United States was $61,650. A recent sample of 36 entry level engineers reveals a mean salary of $69,800 with a standard deviation of $5,000. Using a significance level .02, is this enough proof to conclude that the average salary has increased?

HYPOTHESIS TESTING

Solution 1:

Recall that there are four steps to completing a hypothesis test:

1. State the hypotheses
2. Formulate an analysis plan
3. Analyze sample data
4. Interpret results

So let's run through the steps:

Step 1: State the hypotheses

With the given information, we can state the hypothesis as:

$$H_0 : \mu = 150$$
$$H_A : \mu \neq 150$$

Step 2: Formulate an analysis plan

- The significance level is defined as .01, since this is a double tailed problem, .005 in the right tail, and .005 in the left tail will define the rejection region.
- The testing method will be the one-sample t-test

Step 3: Analyze sample data

- Compute the standard error (SE) of the sampling distribution, which is done with the equation:

$$SE = \frac{s}{\sqrt{n}} = \frac{30.20}{\sqrt{40}} = 4.78$$

- Determine the degrees of freedom (DF), which is equal to the sample size minus one or:

$$DF = n - 1 = 40 - 1 = 39$$

- Define the test statistic, a t-score, defined as:

$$t = \frac{(\overline{x} - \mu)}{SE} = \frac{(137 - 150)}{4.78} = -2.72$$

Where $\overline{x}$ is the sample mean, μ is the hypothesized population mean in the null hypothesis, and SE is the standard error.

- Finally, determine the P-value using the t-distribution table. The p-value is the probability of observing a sample statistic as extreme as the test statistic. This is a double tail problem where a t-score of -2.72 and 39 degrees of freedom corresponds to a p-value of approximately .0045

Step 4: Interpret results

- Comparing the P-value to the significance level, we find that .0045< .005, which calls for the rejection of the null hypothesis. It can be concluded that the mean sales at this shop is not $150.

Solution 2:

Recall that there are four steps to completing a hypothesis test:

1. State the hypotheses
2. Formulate an analysis plan
3. Analyze sample data
4. Interpret results

So let's run through the steps:

Step 1: State the hypotheses

With the given information, we can state the hypothesis as:

$$H_0 : \mu = 15$$
$$H_A : \mu < 15$$

Step 2: Formulate an analysis plan

- The significance level is defined as .025
- The testing method will be the one-sample t-test

Step 3: Analyze sample data

- Compute the standard error (SE) of the sampling distribution, which is done with the equation:

$$SE = \frac{s}{\sqrt{n}} = \frac{3.6}{\sqrt{16}} = .9$$

- Determine the degrees of freedom (DF), which is equal to the sample size minus one or:

$$DF = n - 1 = 16 - 1 = 15$$

- Define the test statistic, a t-score, defined as:

$$t = \frac{(\bar{x} - \mu)}{SE} = \frac{(13.5 - 15)}{.9} = -1.67$$

Where $\bar{x}$ is the sample mean, μ is the hypothesized population mean in the null hypothesis, and SE is the standard error.

- Finally, determine the P-value using the t-distribution table. The p-value is the probability of observing a sample statistic as extreme as the test statistic. This is a single tail problem where a t-score of -1.67 and 15 degrees of freedom corresponds to a p-value of approximately .06

Step 4: Interpret results

- Comparing the P-value to the significance level, we find that .06> .025, which means that we can't reject the null hypothesis, the editor's claim is true.

Solution 3:

At one point in time, it was determined that the mean salary for an entry level engineer in the United States was $61,650. A recent sample of 36 entry level engineers reveals a mean salary of $63,800 with a standard deviation of $5,000. Using a significance level .02, is this enough proof to conclude that the average salary has increased?
Let's run through the steps:

Step 1: State the hypotheses

With the given information, we can state the hypothesis as:

$$H_0 : \mu = 61{,}650$$
$$H_A : \mu > 61{,}650$$

Step 2: Formulate an analysis plan

- The significance level is defined as .02
- The testing method will be the one-sample t-test

Step 3: Analyze sample data

- Compute the standard error (SE) of the sampling distribution, which is done with the equation:

$$SE = \frac{s}{\sqrt{n}} = \frac{5000}{\sqrt{36}} = 833$$

- Determine the degrees of freedom (DF), which is equal to the sample size minus one or:

$$DF = n - 1 = 36 - 1 = 35$$

- Define the test statistic, a t-score, defined as:

$$t = \frac{(\overline{x} - \mu)}{SE} = \frac{(63{,}800 - 61{,}650)}{833} = 2.58$$

Where $\overline{x}$ is the sample mean, μ is the hypothesized population mean in the null hypothesis, and SE is the standard error.

- Finally, determine the P-value using the t-distribution table. The p-value is the probability of observing a sample statistic as extreme as the test statistic. This is a single tail problem where a t-score of 2.58 and 35 degrees of freedom corresponds to a p-value of approximately .008

Step 4: Interpret results

- Comparing the P-value to the significance level, we find that .008< .02, which calls for the rejection of the null hypothesis. It can be concluded that the mean salary is now greater than $61,650.