CONFIDENCE INTERVALS

A confidence interval can be defined after determining three pieces of information.

- Confidence level
- Statistic
- Margin of error

With these inputs, the range of the confidence interval is defined by:

$$Sample\ Statistic \pm Margin\ of\ Error$$

With the uncertainty associated with the confidence interval being specified by the confidence level.

There are four steps to defining a confidence interval.

- Identify a sample statistic that will be used to estimate a population parameter, usually a mean or proportion.
- Select a confidence level describing the uncertainty of the sampling method. Often, a 90, 95, or 99% confidence level is chosen, though any percentage can be used.
- Find the margin of error calculated using the following equation:
  Margin of error (ME) = Critical value x Standard error of statistic
- Define the confidence interval using the following equation:
  Confidence interval = sample statistic $\pm$ Margin of error
  The uncertainty is denoted by the confidence level.

There are some definitions to note when calculating the Marin of Error, the first being the standard error. The standard error is computed from known sample statistics, providing an estimate of the standard deviation. Given a certain statistic, the most common formulas used to calculate the standard error are:

Statistic	Standard Error
Sample Mean, $\bar{x}$	$SE = \frac{s}{\sqrt{n}}$
Proportion, p	$SE = \sqrt{\frac{p(1-p)}{n}}$
Difference between Means, $\bar{x}_1 - \bar{x}_2$	$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
Difference between Proportions, $p_1 - p_2$	$SE = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$

To determine the critical value, the following steps are used:

- Compute alpha, α = 1 - (confidence level / 100)
- Find the critical probability, $p^* = 1 - \frac{\alpha}{2}$
- Express the critical value as a t score by:
  - Finding the degrees of freedom (DF), typically equal to the sample size minus one.
  - The critical t score, t^*, is the t score having degrees of freedom equal to DF and a cumulative probability equal to the critical probability, p^*

Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

A sample of 16 students is taken. The average age in the sample is 22 years old with a standard deviation of 6 years. Determine the 95% confidence interval for the average age of the population.

Solution:

There are four steps to defining a confidence interval.

- Identify a sample statistic that will be used to estimate a population parameter, usually a mean or proportion.

    This problem deals with a sample mean, $\bar{x} = 22$

- Select a confidence level describing the uncertainty of the sampling method.

    The problem requests a 95% confidence interval

- Find the margin of error calculated using the following equation:

    Determine the Critical value:

    - Compute alpha, α = 1 - (95 / 100)=.05
    - Find the critical probability, $p^* = 1 - \frac{.05}{2} = .975$
    - Express the critical value as a t score by:
        - Finding the degrees of freedom (DF), 16-1=15
        - The critical t score, t^*, is the t score having degrees of freedom equal to DF and a cumulative probability equal to the critical probability, p^*, 2.131

    Determine the Standard Error:

    $$SE = \frac{6}{\sqrt{16}} = 1.5$$

    The Margin of error (ME) = 2.131(1.5)=3.2

- The 95% confidence interval is then 22 ± 3.2

It's important to note that this problem could have been done using the z-score along with the Normal Distribution tables, but would result in a slightly different answer and a tighter interval. However, when a sample size is small (below 30) it is more appropriate to use the t-score and the t-distribution tables.