T DISTRIBUTIONS

According to the central limit theorem, the sampling distribution of a statistic will follow a normal distribution, as long as the sample size is sufficiently large. Therefore, when the standard deviation of a population is known, the z-score and normal distribution can be used to evaluate probabilities with the sample mean.

When sample sizes are small or the standard deviation of the population is not known, one can use the t score given by the equation:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Where $\bar{x}$ is the sample mean, μ is the population mean, s is the standard deviation of the sample, and n is the sample size. The distribution of the *t* score is called the t distribution.

The form of the t distribution is determined by its degrees of freedom. The degrees of freedom refers to the number of independent observations in a set of data.

When estimating a mean score, the degrees of freedom is equal to the sample size minus one or:

$$DF = n - 1$$

The t distribution has the following properties:

- The mean of the distribution is equal to 0.
- The variance is equal to

$$\frac{v}{(v-2)}$$

Where *v* is the degrees of freedom.

- The variance is always greater than 1, although it is close to 1 when there are many degrees of freedom.

It's important to note that a t distribution with infinite degrees of freedom is the same as the standard normal distribution.

The t distribution can be used with any statistic that is approximately normal and should not be used with small samples from populations that are not approximately normal.

When a t score is produced, it can be associated with a unique cumulative probability found in the t distribution tables. This cumulative probability represents the likelihood of finding a sample mean less than or equal to $\overline{x}$, given a random sample of size *n*.

Often times t_{α} is used to represent the t-score that has a cumulative probability of $(1-\alpha)$. As an example, if we are interested in the t-score having a cumulative probability of 0.95, α would be equal to (1 - 0.95) or 0.05. and the t-score would be referred to as $t_{.05}$. The value of $t_{.05}$ depends on the number of degrees of freedom

It's important to note that due to the symmetry of the t distribution, the following is true.

$$t_{\alpha} = -t_{1-\alpha} \text{ and } t_{1-\alpha} = -t_{\alpha}$$

Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

A certain Company claims that their repelling rope has an average breaking strength of 20,000 pounds, with a standard deviation of 1750 pounds. Suppose a customer tests

14 randomly selected ropes, determine the probability that the average breaking strength in the test will be no more than 19,800 pounds.

Solution:

The approach to solving this problem follows a two-step process as follows:

- Compute a t score, assuming that the mean of the sample test is 19,800 pounds.
- Determine the cumulative probability for that t score.

The first step involves calculating the t score using the following equation:

$$t = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Where $\overline{x}$ is the sample mean (19,800), μ is the population mean (20,000), s is the standard deviation of the sample (1750), and n is the sample size (14). Plugging these values in to the equation we get:

$$t = \frac{19800 - 20000}{\frac{1750}{\sqrt{14}}} = -.428$$

The next step is to determinine the cumulative probability for the t score using the t distribution tables. We know:

- $t = -.428$
- $DF = 14 - 1 = 13$

Using the t distribution tables we find that these values correspond to a cumulative probability of .34. Therefore, there is a 34% chance that the average breaking strength of the ropes in the test will be no more than 19,800 pounds.