BINOMIAL DISTRIBUTION AND PROBABILITY

A binomial experiment is a statistical experiment that has the following properties:

- The experiment consists of *n* repeated trials.
- Each trial can result in just two possible outcomes, either a success or a failure.
- The probability of success, denoted by *P*, is the same on every trial.
- The trials are independent, which means the outcome of one trial does not affect the outcome of other trials.
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The following notation is commonly used when talking about binomial probability.

- *x*: The number of successes that result from the binomial experiment.
- *n*: The number of trials in the binomial experiment.
- *P*: The probability of success on an individual trial.
- *Q*: The probability of failure on an individual trial. (This is equal to 1 - *P*.)
- b(*x*; *n, P*): Binomial probability - the probability that an *n*-trial binomial experiment results in exactly *x* successes, when the probability of success on an individual trial is *P*. We will also commonly see this written as Pr(X=x), where little x is the number of successes.
- $_nC_r$: The number of combinations of *n* things, taken *r* at a time.

A binomial random variable is the number of successes *x* in *n* repeated trials of a binomial experiment. The probability distribution of a binomial random variable is called a binomial distribution

The binomial distribution has the following properties:

- The mean of the distribution, E(X) or μ, is equal to *nP*
- The variance, V(X) or σ^2, is nP(1-P)
- The standard deviation σ is $\sqrt{nP(1-P)}$

The binomial probability refers to the probability that a binomial experiment will result in exactly *x* successes.

Suppose a binomial experiment consists of *n* trials and results in *x* successes. If the probability of success on an individual trial is *P*, then the binomial probability can be calculated using the following formula:

$$\Pr(X = x) = b(x;n,P) = {}_nC_x (P^x (1-P)^{n-x}$$

A cumulative binomial probability refers to the probability that the binomial random variable falls within a specified range. As an example, the probability of getting at most 2 tails in a 3 quarter tosses is an example of cumulative probability. It is equal to the probability of getting 0 tails (.125) plus the probability of getting 1 tail (.375) plus the probability of getting 2 tails (.375). Therefore, the cumulative probability of getting at most 2 tails in a 3 coin toss would be equal to .875.

Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

A certain traffic study reveals that 88% of adult drivers do not use seat belts. If ten adult drivers are stopped at random, what will the probability that at least 8 drivers will not be wearing seat belts?

Solution:

This is a cumulative binomial probability problem. The probability that at least 8 drivers will not be wearing seat belts is equal to the probability that 8 drivers will not be wearing a seat belt, plus the probability that 9 drivers will not be wearing a seat belt, plus the probability that 10 drivers will not be wearing a seat belt. Therefore:

$$\begin{aligned} \text{P(at least 8)} &= P(8) + P(9) + P(10) \\ &= {}_{10}C_8 \cdot \left(0.88\right)^8 \cdot (0.12)^2 + {}_{10}C_9 \cdot (0.88)^9 \cdot (0.12)^1 + {}_{10}C_{10} \cdot (0.88)^{10} \cdot (0.12)^0 \\ &= 0.23 + 0.38 + 0.28 = 0.89 \end{aligned}$$