COMBINATIONS

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Find the number of ways to take 20 objects and arrange them in groups of 5 at a time where order does not matter.

Problem 2:

How many ways are there to select a committee of 7 members from pool of 17 individuals?

Problem 3:

Determine the total number of 5 card hands that can be drawn from a deck of 52 cards.

COMBINATIONS

Solution 1:

Since order does not matter, we are dealing with a combination problem.

Recall that a combination of n objects taken r at a time is an arrangement of *r* objects, without regard to order and without repetition, selected from *n* distinct objects, given by the formula:

$$_{n}C_{r} = \frac{n!}{(n-r)!r!}$$

From the problem we can define the following:

$$n = 20$$
$$r = 5$$

Plugging these values in to the equation for a combination:

$$_{4}C_{3} = \frac{20!}{(20-5)!5!} = 15{,}504$$

Therefore, there are 15,504 ways to arrange 20 objects taken 5 at a time when order does not matter.

Solution 2:

Since order does not matter, we are dealing with a combination problem. Committees are always a combination unless the problem states that there is a set hierarchy of the members within the committee (ie, president, vice-presidents, etc). If a committee is ordered in a hierarchy, then it is a permutation.

Recall that a combination of n objects taken r at a time is an arrangement of *r* objects, without regard to order and without repetition, selected from *n* distinct objects, given by the formula:

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

From the problem we can define the following:

$$n = 17$$
$$r = 7$$

Plugging these values in to the equation for a combination:

$${}_4C_3 = \frac{17!}{(17-7)!7!} = 19{,}448$$

Therefore, there are 19,448 different ways to form the committee.

Solution 3:

When a deck of cards is dealt, order does not matter, and since order does not matter, we are dealing with a combination problem.

Recall that a combination of n objects taken r at a time is an arrangement of *r* objects, without regard to order and without repetition, selected from *n* distinct objects, given by the formula:

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

From the problem we can define the following:

$$n = 52$$

$r = 5$

Plugging these values in to the equation for a combination:

$$_{4}C_{3} = \frac{52!}{(52-5)!5!} = 2,598,960$$

Therefore, there are 2,598,960 ways five cards can be drawn from a deck of cards…that's a lot.