POWER SERIES

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Determine the interval of convergence for the series $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n \cdot 5^n}$

Problem 2:

Determine the interval of convergence for the series $\sum_{n=0}^{\infty} \frac{2^{2n} x^n}{n^2}$

Problem 3:

Determine where the series $\sum_{n=0}^{\infty} \frac{(x+3)^n}{n+1}$ diverges

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Solution 1:

Recall that the most useful test for convergence of a power series is the ratio test. If $c_n \neq 0$, and if for a fixed value:

$$\lim_{n\to\infty}\left|\frac{c_{n+1}(x-a)^{n+1}}{c_n(x-a)^n}\right| = |x-a|\lim_{n\to\infty}\left|\frac{c_{n+1}}{c_n}\right| = |x-a|L$$

Then the power series converges at that value of x if $|x-a|L<1$ and diverges if $|x-a|L>1$. If $|x-a|L=1$, then the test is inconclusive.

Analyzing the series:

$$\lim_{n\to\infty}\left|\frac{c_{n+1}(x-a)^{n+1}}{c_n(x-a)^n}\right| = \lim_{n\to\infty}\left|\frac{\frac{(x-2)^{n+1}}{(n+1)\cdot 5^{n+1}}}{\frac{(x-2)^n}{n\cdot 5^n}}\right| = |x-2|\lim_{n\to\infty}\left|\frac{n\cdot 5^n}{(n+1)\cdot 5^{n+1}}\right| = |x-2|\frac{1}{5}$$

This tells us that the power series converges for the values of x where $|x-2|\frac{1}{5}<1$ and diverges for the values of x where $|x-2|\frac{1}{5}>1$. To determine the interval of convergence, we solve the convergence inequality and find that the power series converges on the interval:

$|x-2|<5$ or $-3<x<7$

Solution 2:

Analyze the series noting the rules of the ratio test:

$$\lim_{n\to\infty}\left|\frac{c_{n+1}(x-a)^{n+1}}{c_n(x-a)^n}\right| = \lim_{n\to\infty}\left|\frac{\frac{2^{2n+2}x^{n+1}}{(n+1)^2}}{\frac{2^{2n}x^n}{n^2}}\right| = |x|\lim_{n\to\infty}\left|\frac{n^2\cdot 2^{2n+2}}{(n+1)^2\cdot 2^{2n}}\right| = |x|4$$

This tells us that the power series converges for the values of x where $|x|4<1$ and diverges for the values of x where $|x|4>1$. To determine the interval of convergence, we solve the convergence inequality and find that the power series converges on the interval:

$$|x|<\frac{1}{4} \text{ or } -\frac{1}{4}<x<\frac{1}{4}$$

Solution 3:

Recall that the most useful test for convergence of a power series is the ratio test. If $c_n \neq 0$, and if for a fixed value:

$$\lim_{n\to\infty}\left|\frac{c_{n+1}(x-a)^{n+1}}{c_n(x-a)^n}\right| = |x-a|\lim_{n\to\infty}\left|\frac{c_{n+1}}{c_n}\right| = |x-a|L$$

Then the power series converges at that value of x if $|x-a|L<1$ and diverges if $|x-a|L>1$. If $|x-a|L=1$, then the test is inconclusive.

Analyzing the series:

$$\lim_{n\to\infty}\left|\frac{c_{n+1}(x-a)^{n+1}}{c_n(x-a)^n}\right| = \lim_{n\to\infty}\left|\frac{\frac{(x+3)^{n+1}}{n+2}}{\frac{(x+3)^n}{n+1}}\right| = |x+3|\lim_{n\to\infty}\left|\frac{n+1}{n+2}\right| = |x+3|$$

This tells us that the power series converges for the values of x where $|x+3|<1$ and diverges for the values of x where $|x+3|>1$. To determine the interval of convergence, we solve the convergence inequality and find that the power series converges on the interval:

$|x+3|<1$ or -4<x<-2

So the series diverges for $x<-4$ and $x>-2$