GEOMETRIC SEQUENCES AND SERIES

A geometric sequence is a sequence where each consecutive term is derived from the previous term multiplied by a fixed number called a common ratio. The pattern is that we are always multiplying by a fixed number to the previous term to get to the next term.

Not every sequence that has a pattern in multiplication is geometric. It is considered a geometric sequence only if it is being multiplied by the SAME number each and every time.

The general formula for a geometric sequence is:

$$a_n = a_1 r^{n-1}$$

where a_1 is the first term of the sequence and r is the common ratio.

A Geometric series is simply the sum of the first n terms of finite geometric sequence, given by the general formula:

The sum of the first n terms of a finite geometric sequence is:

$$S_n = \frac{a_1(1-r^n)}{1-r};\ r \neq 0$$

where a_1 is the first term of the sequence and r is the common ratio.

Simple math can be used to determine the quantity of any series.

Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

Find the tenth term and the n-th term of the following sequence:

$$\frac{1}{2}, 1, 2, 4, 8\ldots$$

Solution:

The differences do not match, for example, $2 - 1 = 1$, but $4 - 2 = 2$. So this isn't an Arithmetic Sequence. On the other hand, the ratios are the same: $2 \div 1 = 2$, $4 \div 2 = 2$, $8 \div 4 = 2$. So this is a Geometric Sequence with common ratio $r = 2$ and $a_1 = \frac{1}{2}$. To determine the tenth and n^{th} terms, recall that The general formula for a geometric sequence is:

$$a_n = a_1 r^{n-1}$$

where a_1 is the first term of the sequence and r is the common ratio; both which are defined. Plugging these values in to the equation we find that the n^{th} term is:

$$a_n = \frac{1}{2}(2)^{n-1}$$

The 10^{th} term is (plugging in n=10)

$$a_{10} = \frac{1}{2}(2)^{10-1} = \frac{1}{2}(2)^9 = 256$$