SIMULTANEOUS LINEAR EQUATIONS

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Solve the system of equations:

$$x + 2y - z = 6$$
$$3x + 5y - z = 2$$
$$-2x - y - 2z = 4$$

Problem 2:

Determine the currents within an electrical circuit represented by the following equations.:

$$I_A + I_B + I_C = 0$$
$$2I_A - 5I_B = 6$$
$$5I_B - I_C = -3$$

Problem 3:

We want 10 L of gas containing 2% additive and have drums of the following:

- gasoline without additive
- gasoline with 5% additive
- gasoline with 6% additive

We need to use 4 times as much pure gasoline as 5% additive gasoline. Determine much of each is needed.

SIMULTANEOUS LINEAR EQUATIONS

Solution 1:

The process to solve any set of simultaneous equations is:

Express the set of linear equations compactly in the matrix form $Ax = y$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 5 & -1 \\ -2 & -1 & -2 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad y = \begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix}$$

Premultiply both sides of the equation with the inverse matrix of A, A^{-1}:

$$A^{-1}Ax = A^{-1}y$$

The inverse of A is found by multiplying the adjoint matrix by the determinate. The process is as follows:

Determine the Matrix of Minors, which is:

$$\begin{bmatrix} -11 & -8 & 7 \\ -5 & -4 & 3 \\ 3 & 2 & -1 \end{bmatrix}$$

The Matrix of Cofactors is then:

$$\begin{bmatrix} -11 & 8 & 7 \\ 5 & -4 & -3 \\ 3 & -2 & -1 \end{bmatrix}$$

The Adoint matrix is the transpose of the Matrix of Cofactors, which is:

$$\begin{bmatrix} -11 & 5 & 3 \\ 8 & -4 & -2 \\ 7 & -3 & -1 \end{bmatrix}$$

Now find the determinate of A, which is:

$$|A| = 1(-11) - 2(-8) + (-1)(7) = -2$$

So the inverse is:

$$A^{-1} = \frac{1}{-2}\begin{bmatrix} -11 & 5 & 3 \\ 8 & -4 & -2 \\ 7 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 5.5 & -2.5 & -1.5 \\ -4 & 2 & 1 \\ -3.5 & 1.5 & .5 \end{bmatrix}$$

Therefore,

$$x = A^{-1}y$$

$$x = \begin{bmatrix} 5.5 & -2.5 & -1.5 \\ -4 & 2 & 1 \\ -3.5 & 1.5 & .5 \end{bmatrix}\begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix}$$

$$x = \begin{bmatrix} 22 \\ -16 \\ -16 \end{bmatrix}$$

The solutions are then $x = 22$, $y = -16$, and $z = -16$

Confirm these answers by plugging them in to the original equation.

$$x + 2 - z = 6$$

$$3x + 5y - z = 2$$

$$-2x - y - 2z = 4$$

Substituting $x = 22$, $y = -16$, and $z = -16$, we get:

$$22 + 2(-16) - (-16) = 6$$

$$3(22) + 5(-16) - (-16) = 2$$

$$-2(22) - (-16) - 2(-16) = 4$$

The solutions check out.

Solution 2:

The process to solve any set of simultaneous equations is:

Express the set of linear equations compactly in the matrix form $Ax = y$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -5 & 0 \\ 0 & 5 & -1 \end{bmatrix} \quad x = \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \quad y = \begin{bmatrix} 0 \\ 6 \\ -3 \end{bmatrix}$$

Premultiply both sides of the equation with the inverse matrix of A, A^{-1}:

$$A^{-1}Ax = A^{-1}y$$

The inverse of A is found by multiplying the adjoint matrix by the determinate. The process is as follows:

Determine the Matrix of Minors, which is:

$$\begin{bmatrix} 5 & -2 & 10 \\ -6 & -1 & 5 \\ 5 & -2 & -7 \end{bmatrix}$$

The Matrix of Cofactors is then:

$$\begin{bmatrix} 5 & 2 & 10 \\ 6 & -1 & -5 \\ 5 & 2 & -7 \end{bmatrix}$$

The Adoint matrix is the transpose of the Matrix of Cofactors, which is:

$$\begin{bmatrix} 5 & 6 & 5 \\ 2 & -1 & 2 \\ 10 & -5 & -7 \end{bmatrix}$$

Now find the determinate of A, which is:

$$|A| = 1(5) - 1(-2) + 1(10) = 17$$

So the inverse is:

$$A^{-1} = \frac{1}{17}\begin{bmatrix} 5 & 6 & 5 \\ 2 & -1 & 2 \\ 10 & -5 & -7 \end{bmatrix} = \begin{bmatrix} .294 & .353 & .294 \\ .118 & -.059 & .118 \\ .588 & -.294 & -.412 \end{bmatrix}$$

Therefore,

$$x = A^{-1}y$$

$$x = \begin{bmatrix} .294 & .353 & .294 \\ .118 & -.059 & .118 \\ .588 & -.294 & -.412 \end{bmatrix}\begin{bmatrix} 0 \\ 6 \\ -3 \end{bmatrix}$$

$$x = \begin{bmatrix} 1.236 \\ -.708 \\ -.528 \end{bmatrix}$$

The solutions are then $I_A = 1.236A$, $I_B = -.708A$, and $I_C = -.528A$

 psst…don't forget to take notes

Solution 3:

This is a word problem where we need to create our set of linear equations. Let's revisit the problem statement.

We want 10 L of gas containing 2% additive and have drums of the following:

- gasoline without additive
- gasoline with 5% additive
- gasoline with 6% additive

We need to use 4 times as much pure gasoline as 5% additive gasoline. Determine much of each is needed.

Let define the following variables:

x = amount of liters of pure gas
y = amount of liters of 5% gas
z = amount of liters of 6% gas

So the first sentence gives us the equation:

$$x + y + z = 10$$

The next information tells us that:

We get no additive from the first drum
We get 5% additive per liter from the second drum
We get 6% additive per liter from the third drum

We want 2% total additive in 10 L of gas so our second equation is:

$$.05y + .06z = .2$$

Multiplying through by 100 we get:

$$5y + 6z = 20$$

Finally, the last bit of information tells us that we need to use 4 times as much pure gasoline as 5% additive gasoline, so:

$$x = 4y$$

Or

$$x - 4y = 0$$

Now we have our set of linear equations and can run through the process of solving. The equations are:

$$x + y + z = 10$$

$$5y + 6z = 20$$
$$x - 4y = 0$$

The process to solve any set of simultaneous equations is:

Express the set of linear equations compactly in the matrix form $Ax = y$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 5 & 6 \\ 1 & -4 & 0 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad y = \begin{bmatrix} 10 \\ 20 \\ 0 \end{bmatrix}$$

Premultiply both sides of the equation with the inverse matrix of A, A^{-1}:

$$A^{-1}Ax = A^{-1}y$$

The inverse of A is found by multiplying the adjoint matrix by the determinate. The process is as follows:

Determine the Matrix of Minors, which is:

$$\begin{bmatrix} 24 & -6 & -5 \\ 4 & -1 & -5 \\ -1 & 6 & 5 \end{bmatrix}$$

The Matrix of Cofactors is then:

$$\begin{bmatrix} 24 & 6 & -5 \\ -4 & -1 & 5 \\ -1 & -6 & 5 \end{bmatrix}$$

The Adoint matrix is the transpose of the Matrix of Cofactors, which is:

$$\begin{bmatrix} 24 & -4 & -1 \\ 6 & -1 & -6 \\ -5 & 5 & 5 \end{bmatrix}$$

Now find the determinate of A, which is:

$$|A| = 1(24) - 1(-6) + 1(-5) = 25$$

So the inverse is:

$$A^{-1} = \frac{1}{25}\begin{bmatrix} 24 & -4 & -1 \\ 6 & -1 & -6 \\ -5 & 5 & -5 \end{bmatrix} = \begin{bmatrix} .96 & -.16 & -.04 \\ .24 & -.04 & -.24 \\ -.2 & .2 & .2 \end{bmatrix}$$

Therefore,

$$x = A^{-1}y$$

$$x = \begin{bmatrix} .96 & -.16 & -.04 \\ .24 & -.04 & -.24 \\ -.2 & .2 & .2 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 0 \end{bmatrix}$$

$$x = \begin{bmatrix} 6.4 \\ 1.6 \\ 2 \end{bmatrix}$$

So if we want 10 L of gas containing 2% additive and we need to use 4 times as much pure gasoline as 5% additive gasoline, then we need:

- 6.4 L of gasoline without additive
- 1.6 L of gasoline with 5% additive
- 2 L of gasoline with 6% additive