CROSS PRODUCT

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Given the two vectors $\vec{a} = < 1, -1, 0 >$ and $\vec{b} = < 0, -2, 0 >$, determine the angle between them if the cross product $\vec{a} \times \vec{b} = 0\vec{i} - 0\vec{j} - 2\vec{k}$

Problem 2:

Given the two vectors $v = 2i + 5j + k$ and $u = -3i + 2j + 4k$ determine the cross product $v \times a$

Problem 3:

Evaluate $A \times B$ if $A = < 2, -3, 0 >$ and $B = < 3, 1, -4 >$

CROSS PRODUCT

Solution 1:

Given two vectors $\vec{a} =< a_1, a_2, a_3 >$ and $\vec{b} =< b_1, b_2, b_3 >$ with an angle Θ between them, where Θ is between 0 and π, the cross product can be found using the following formula:

$$\left|\vec{a}\times\vec{b}\right| = \left|\vec{a}\right|\left|\vec{b}\right|\sin\Theta$$

In this problem we are given the cross product, along with the individual vectors so all that needs to be done is to determine the values of the magnitudes.

$$\left|\vec{a}\right| = \sqrt{1^2 + (-1^2) + 0^2} = \sqrt{2}$$

$$\left|\vec{b}\right| = \sqrt{0^2 + (-2)^2 + 0^2} = 2$$

$$\left|\vec{a}\times\vec{b}\right| \sqrt{0^2 + 0^2 + (-2)^2} = 2$$

Plugging these values in to the equation:

$$2 = \sqrt{2}(2)\sin\Theta$$

Rearranging and solving for the angle we get:

$$\Theta = \sin^{-1}\frac{2}{\sqrt{2}(2)} = 45^\circ$$

Solution 2:

The cross product can be found by first inserting the two vectors in to a 3x3 matrix such that:

$$v \times a = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 5 & 1 \\ -3 & 2 & 4 \end{vmatrix}$$

Using the Method of Cofactors, the cross product is:

$$v \times a = \begin{vmatrix} 5 & 1 \\ 2 & 4 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & 1 \\ -3 & 4 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & 5 \\ -3 & 2 \end{vmatrix} \vec{k} \text{ where } \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

The cross product of the two vectors is then:

$$v \times a = 18\vec{i} - 11\vec{j} + 19\vec{k}$$

Solution 3:

The cross product can be found by first inserting the two vectors in to a 3x3 matrix such that:

$$A \times B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 0 \\ 3 & 1 & -4 \end{vmatrix}$$

Using the Method of Cofactors, the cross product is:

$$A \times B = \begin{vmatrix} -3 & 0 \\ 1 & -4 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & 0 \\ 3 & -4 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & -3 \\ 3 & 1 \end{vmatrix} \vec{k} \text{ where } \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

The cross product of the two vectors is then:

$$A \times B = 12\vec{i} + 8 + 11\vec{k}$$