DOT PRODUCT

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Compute the dot product given the two vectors $\vec{a} = <0,0,3>$ and $\vec{b} = <2,0,2>$ with an angle of 45° between them.

Problem 2:

Compute the dot product given the two vectors $\vec{a} = <0,2,-1>$ and $\vec{b} = <-1,1,2>$ with an angle of 90° between them.

Problem 3:

Determine the angle between the two vectors $\vec{a} = <2,5,0>$ and $\vec{b} = <-3,2,0>$

DOT PRODUCT

Solution 1:

Given two vectors $\vec{a} =< a_1, a_2, a_3 >$ and $\vec{b} =< b_1, b_2, b_3 >$ with an angle Θ between them, where Θ is between 0 and π, the dot product can be found using the following formula:

$$\left|\vec{a} \bullet \vec{b}\right| = |\vec{a}|\left|\vec{b}\right| \cos\Theta$$

Therefore, defining the necessary values:

$$|\vec{a}| = \sqrt{0+0+9} = 3$$
$$\left|\vec{b}\right| = \sqrt{4+0+4} = 2\sqrt{2}$$

And knowing the angle is 45°, plug in the values to determine the dot product:

$$\left|\vec{a} \bullet \vec{b}\right| = 3 \cdot 2\sqrt{2}\cos(45) = 6$$

Solution 2:

Again, given two vectors $\vec{a} =< a_1, a_2, a_3 >$ and $\vec{b} =< b_1, b_2, b_3 >$ with an angle Θ between them, where Θ is between 0 and π, the dot product can be found using the following formula:

$$\left|\vec{a} \bullet \vec{b}\right| = |\vec{a}|\left|\vec{b}\right| \cos\Theta$$

We could either solve the problem using this formula or the traditional formula.

Therefore, defining the necessary values:

$$|\vec{a}| = \sqrt{0+4+1} = \sqrt{5}$$

$$\left|\vec{b}\right| = \sqrt{1+1+4} = \sqrt{6}$$

And knowing the angle is 90°, plug in the values to determine the dot product:

$$\left|\vec{a} \bullet \vec{b}\right| = \sqrt{5} \cdot \sqrt{6}\cos(90) = 0$$

Solution 3:

Determine the angle between the two vectors $\vec{a} =< 2,5,0 >$ and $\vec{b} =< -3,2,0 >$

This problem is solved similar to the previous, but we need to use both the traditional dot product formula and the $\left|\vec{a} \bullet \vec{b}\right| = \left|\vec{a}\right|\left|\vec{b}\right|\cos\Theta$ to determine the angle.

Therefore, recall that when given two vectors $\vec{a} =< a_1, a_2, a_3 >$ and $\vec{b} =< b_1, b_2, b_3 >$, the dot product is found using the following formula:

$$\vec{a} \bullet \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Plugging in the values given:

$$\vec{a} \bullet \vec{b} = 2(-3) + 5(2) + 0(0) = 4$$

Now define the magnitude of each of the vectors:

$$\left|\vec{a}\right| = \sqrt{4+25+0} = \sqrt{29}$$
$$\left|\vec{b}\right| = \sqrt{9+4+0} = \sqrt{13}$$

Plug the values in and solve for Θ:

$$4 = \sqrt{29}\sqrt{13}\cos\Theta$$
$$\Theta = \cos^{-1}\frac{4}{\sqrt{29}\sqrt{13}} = 78.1^{\circ}$$