ELEMENTARY ROW OPERATIONS

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Complete the row operation $R2 \Leftrightarrow R1$ on the following matrix:

$$A = \begin{bmatrix} 1 & 9 \\ 7 & 5 \end{bmatrix}$$

Problem 2:

Complete the column operation $C1 + 5C2$ on the following matrix:

$$A = \begin{bmatrix} 1 & 6 \\ 4 & 3 \\ 6 & 3 \end{bmatrix}$$

Problem 3:

Complete the row operation $R2 - 2R1$ on the following matrix:

$$A = \begin{bmatrix} 2 & 4 & 1 \\ 3 & 4 & 7 \\ 5 & 9 & 3 \end{bmatrix}$$

ELEMENTARY ROW OPERATIONS

Solution 1:

To perform an elementary row operation on an *r* x *c* matrix, the following steps are completed:

1. Find E, the elementary row operator, by applying the desired operation to an *r* x *r* identity matrix.

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad R2 \Leftrightarrow R1 \rightarrow E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

2. Carry out the elementary row operation by premultiplying A by E.

$$EA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 9 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 0(1)+1(7) & 0(9)+1(5) \\ 1(1)+0(7) & 1(9)+0(5) \end{bmatrix}$$

Therefore, the row operation $R2 \Leftrightarrow R1$ on $A = \begin{bmatrix} 1 & 9 \\ 7 & 5 \end{bmatrix}$ is:

$$EA = \begin{bmatrix} 7 & 5 \\ 1 & 9 \end{bmatrix}$$

Solution 2:

To perform an elementary column operation on an *r* x *c* matrix, the following steps are completed:

1. Find E, the elementary column operator, by applying the operation to an *c* x *c* identity matrix.

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad C1 + 5C2 \Rightarrow E = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

2. Carry out the elementary column operation by postmultiplying A by E.

$$AE = \begin{bmatrix} 1 & 6 \\ 4 & 3 \\ 6 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 1(1)+6(5) & 1(0)+6(1) \\ 4(1)+3(5) & 4(0)+3(1) \\ 6(1)+3(5) & 6(0)+3(1) \end{bmatrix}$$

Therefore, the column operation $C1 + 5C2$ on $A = \begin{bmatrix} 1 & 6 \\ 4 & 3 \\ 6 & 3 \end{bmatrix}$ is:

$$AE = \begin{bmatrix} 31 & 6 \\ 19 & 3 \\ 21 & 3 \end{bmatrix}$$

Solution 3:

To perform an elementary row operation on an *r* x *c* matrix, the following steps are completed:

3. Find E, the elementary row operator, by applying the desired operation to an *r* x *r* identity matrix.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R2 - 2R1 \Rightarrow E = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Carry out the elementary row operation by premultiplying A by E.

$$EA = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 1 \\ 3 & 4 & 7 \\ 5 & 9 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1(2)+0(3)+0(5) & 1(4)+0(4)+0(9) & 1(1)+0(7)+0(3) \\ -2(2)+1(3)+0(5) & -2(4)+1(4)+0(9) & -2(1)+1(7)+0(3) \\ 0(2)+0(3)+1(5) & 0(4)+0(4)+1(9) & 0(1)+0(7)+1(3) \end{bmatrix}$$

Therefore, the row operation $R2-2R1$ on $A = \begin{bmatrix} 2 & 4 & 1 \\ 3 & 4 & 7 \\ 5 & 9 & 3 \end{bmatrix}$ is:

$$EA = \begin{bmatrix} 2 & 4 & 1 \\ -1 & -4 & 5 \\ 5 & 9 & 3 \end{bmatrix}$$