# MATRIX TRANSPOSE AND DETERMINATE

The transpose of a matrix is completed simply by taking the rows in the original matrix and turning them in to columns in a new matrix, or the transposed matrix.

To illustrate the process:

Given the Matrix A:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

Then the transpose of this matrix is:

$$A^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix}$$

The Determinate of a matrix is a real number associated with every square matrix. The determinant of a square matrix A is denoted by either "det A" or |A|.

To illustrate the process:

Given the Matrix A:

$A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ the determinate is $|A| = a_{11}a_{22} - a_{12}a_{21}$

It is important to note that the square brackets do not mean an absolute value, rather, that the value is the determinate of the defined Matrix. A determinate can be a negative number.

To find the Determinate of a larger square matrix, the Method of Cofactors can be used.

Given the Matrix A:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

This method requires that we choose an element, minor, starting in the upper left corner, first row of the first column. After ignoring the other elements (picture drawing a line through them) within that row and column, we multiply the minor by the determinate of the remaining elements; a value is obtained. This process is continued across the top row, rotating plus and minus signs between the values. At the end, the values are combined to create one value, known as the determinate.

This process can be illustrated as such:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$|A|=a_{11}\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}-a_{12}\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}+a_{13}\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

Now the process is simplified to finding the individual determinates of the 2x2 matrices and multiplying them by their respected minor.

Once again, it's important to note the rotation of the plus and minus sign between the values.

## Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

Determine the transpose of the follow matrix:

$$A = \begin{bmatrix} 2 & 0 & 4 & 7 & 54 \\ 45 & 1 & 0 & 3 & 9 \\ 6 & 4 & 8 & 7 & 41 \end{bmatrix}$$

## Solution:

Recall that when given a Matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

Then the transpose of this matrix is:

$$A^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix}$$

Therefore:

$$A = \begin{bmatrix} 2 & 0 & 4 & 7 & 54 \\ 45 & 1 & 0 & 3 & 9 \\ 6 & 4 & 8 & 7 & 41 \end{bmatrix}$$

And

$$A^T = \begin{bmatrix} 2 & 45 & 6 \\ 0 & 1 & 4 \\ 4 & 0 & 8 \\ 7 & 3 & 7 \\ 54 & 9 & 41 \end{bmatrix}$$