MATRIX TRANSPOSE AND DETERMINATE

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Determine the transpose of the follow matrix:

$$A = \begin{bmatrix} 1 & 5 & 7 \\ 5 & 3 & 2 \\ 7 & 2 & 0 \end{bmatrix}$$

Problem 2:

Find the determinate of the following matrix:

$$A = \begin{bmatrix} 2 & -2 & 0 \\ -1 & 5 & 1 \\ 3 & 4 & 5 \end{bmatrix}$$

Problem 3:

Find the determinate of the following matrix:

$$A = \begin{bmatrix} 7 & 4 & 2 & 0 \\ 6 & 3 & -1 & 2 \\ 4 & 6 & 2 & 5 \\ 8 & 2 & -7 & 1 \end{bmatrix}$$

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Solution 1:

Recall that when given a Matrix:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

Then the transpose of this matrix is:

$$A^T=\begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix}$$

Therefore:

$$A=\begin{bmatrix} 2 & -2 & 0 \\ -1 & 5 & 1 \\ 3 & 4 & 5 \end{bmatrix}$$

And

$$A^T=\begin{bmatrix} 1 & 5 & 7 \\ 5 & 3 & 2 \\ 7 & 2 & 0 \end{bmatrix}$$

Solution 2:

To find the Determinate of a 3X3 square matrix, the Method of Cofactors is used.

Given the Matrix A:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Recall that this method requires that we choose an element, minor, starting in the upper left corner, first row of the first column. After ignoring the other elements (picture drawing a line through them) within that row and column, we multiply the minor by the determinate of the remaining elements; a value is obtained. This process is continued across the top row, rotating plus and minus signs between the values. At the end, the values are combined to create one value, known as the determinate.

This process can be illustrated as such:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$|A|=a_{11}\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}-a_{12}\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}+a_{13}\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

Therefore, given:

$$A=\begin{bmatrix} 2 & -2 & 0 \\ -1 & 5 & 1 \\ 3 & 4 & 5 \end{bmatrix}$$

The determinate is:

$$|A|=2\begin{bmatrix} 5 & 1 \\ 4 & 5 \end{bmatrix}-(-2)\begin{bmatrix} -1 & 1 \\ 3 & 5 \end{bmatrix}+0\begin{bmatrix} -1 & 5 \\ 3 & 4 \end{bmatrix}=2(21)-(-2)(-8)+0$$

Or

$$|A| = 26$$

Solution 3:

To find the Determinate of a 4x4 square matrix, the Method of Cofactors is used. However, when dealing with matrices larger than 3x3, multiple instances of the Method of Cofactors is used. In this problem, we first need to break down the 4x4 matrix in to a set of cofactors and 3x3 matrices. We follow that up with breaking down the 3x3 matrices using the Method of Cofactors again to reach the final result.

Therefore, given:

$$A = \begin{bmatrix} 7 & 4 & 2 & 0 \\ 6 & 3 & -1 & 2 \\ 4 & 6 & 2 & 5 \\ 8 & 2 & -7 & 1 \end{bmatrix}$$

The determinate is:

$$|A| = 7\begin{bmatrix} 3 & -1 & 2 \\ 6 & 2 & 5 \\ 2 & -7 & 1 \end{bmatrix} - 4\begin{bmatrix} 6 & -1 & 2 \\ 4 & 2 & 5 \\ 8 & -7 & 1 \end{bmatrix} + 2\begin{bmatrix} 6 & 3 & 2 \\ 4 & 6 & 5 \\ 8 & 2 & 1 \end{bmatrix} - 0\begin{bmatrix} 6 & 3 & -1 \\ 4 & 6 & 2 \\ 8 & 2 & -7 \end{bmatrix}$$

Using the Method of Cofactors again on each of the 3x3 matrices:

$$\begin{bmatrix} 3 & -1 & 2 \\ 6 & 2 & 5 \\ 2 & -7 & 1 \end{bmatrix} = 15$$

$$\begin{bmatrix} 6 & -1 & 2 \\ 4 & 2 & 5 \\ 8 & -7 & 1 \end{bmatrix} = 156$$

$$\begin{bmatrix} 6 & 3 & 2 \\ 4 & 6 & 5 \\ 8 & 2 & 1 \end{bmatrix} = 54$$

$$\begin{bmatrix} 6 & 3 & -1 \\ 4 & 6 & 2 \\ 8 & 2 & -7 \end{bmatrix} = -42$$

Now plugging these values back in to the original breakdown, we get:

$$|A| = 7(15) - 4(156) + 2(54) - 0(-42)$$

So:

$$|A| = -279$$