COMPLEX NUMBERS

The standard form of a complex number is $a + bi$, where a and b are real numbers and can be anything, positive, negative, zero, fractions, decimals, etc. The a is called the real part of the complex number and b is called the imaginary part of the complex number.

It's not possible to get a real number out of a square root of a negative number. For example, $\sqrt{-n}$ isn't a real number since there is no real number that can be squared to get a negative number. However, this number can be reduced and rewritten as:

$$\sqrt{n}\sqrt{-1}$$

Once a number is in this reduced form, the following definition can be used to substitute for the non real number $\sqrt{-1}$.

$$\sqrt{-1} = i \text{ and } i^2 = -1$$

This gives the ability to write $\sqrt{-n}$ as $\sqrt{n}i$, which shows us how we can get to and from the imaginary part of a complex number.

The conjugate of a complex number $a + bi$ is the complex number $a - bi$, which is simply the original complex number with the sign of the imaginary part changed.

The easiest way to think of adding and/or subtracting complex numbers is to think of each complex number as a polynomial and do the addition and subtraction in the same way as it would be otherwise, imaginary number aside.

When dealing with the multiplication of complex numbers, it is again easiest to just think of complex numbers as polynomials and multiplying them out as done normally, imaginary number aside.

CONCEPT INTRODUCTION

The product of a conjugate pair, $(a+bi)(a-bi)$, leads to a real number result that can be determined quickly using the following formula:

$$(a+bi)(a-bi) = a^2 + b^2$$

Concept Example:

The following problem introduces the concept reviewed within this module. Use this content as a primer for the subsequent material.

Perform the following operation and write the result in standard form:

$$(6-2i)(2-3i)$$

Solution:

Multiplying gives the result:

$$(6-2i)(2-3i) = 12 - 18i - 4i + 6i^2$$

Recall that $i^2 = -1$, which simplifies the result to:

$$12 - 22i - 6$$

Putting the final result in to standard form:

$$(6-2i)(2-3i) = 6 - 22i$$