POWER FUNCTIONS

Complete the following problems to reinforce your understanding of the concept covered in this module.

Problem 1:

Simplify the following exponential expression:

$$\left(\frac{x^{m-2n}}{x^{3n}}\right)^{2n} \div \left(\frac{x^{n+m}}{x^{m}}\right)^{-10n}$$

Problem 2:

Simplify and evaluate the following exponential expression:

$$5+8^{-\frac{2}{3}}-81^{-\frac{1}{4}}$$

Problem 3:

Simplify the exponential expression:

$$\frac{625^{x+3}}{5^{4x}(125)+25^{x-1}(5)^{2x-4}}$$

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Solution 1:

The first step in simplifying this expression is to use the quotient rule to simplify both terms within the parenthesis so that:

$$\left(x^{m-5n}\right)^{2n} \div \left(x^{n}\right)^{-10n} = \frac{\left(x^{m-5n}\right)^{2n}}{\left(x^{n}\right)^{-10n}}$$

Use the power rule to get rid of the exponents outside the parenthesis:

$$\frac{x^{2nm-10n^2}}{x^{-10n^2}}$$

Use the multiplication rule to separate the exponents in the numerator so that:

$$\frac{x^{2nm}x^{-10n^2}}{x^{-10n^2}}$$

Finally, cancel out the like terms to get the final answer:

$$x^{2nm}$$

Solution 2:

The first step is to get rid of the negative exponents so that:

$$5+\frac{1}{8^{\frac{2}{3}}}-\frac{1}{81^{\frac{1}{4}}}$$

Next, convert the fractional exponents into their root and power forms:

$$5+\frac{1}{\left(\sqrt[3]{8}\right)^{2}}-\frac{1}{\sqrt[4]{81}}$$

Now evaluate these denominators:

$$5+\frac{1}{\left(2\right)^{2}}-\frac{1}{3}$$

Lastly, evaluating the expression we get:

$$5+\frac{1}{4}-\frac{1}{3}=\frac{59}{12}$$

Solution 3:

The first step is to convert all the values in to a common base exponential function. To do that, we can convert 625, 125, and 25 in to power of 5 exponents so that:

$$\frac{(5^{4})^{x+3}}{5^{4x}\left(5^{3}\right)+(5^{2})^{x-1}\left(5\right)^{2x-4}}$$

Use the power rule to get rid of exponents outside of parentheses:

$$\frac{5^{4x+12}}{5^{4x}\left(5^{3}\right)+5^{2x-2}(5^{2x-4})}$$

Use the multiplication rule to combine terms in the denominator:

$$\frac{5^{4x+12}}{5^{4x+3}+5^{4x-6}}$$

Note that there is a power of 4x in every term, both in the numerator and the denominator. We want to cancel these terms out, so use the multiplication rule in reverse to separate these powers:

$$\frac{5^{4x} \cdot 5^{12}}{5^{4x} \cdot 5^{3} + 5^{4x} \cdot 5^{-6})}$$

Use the distributive property to rewrite the denominator:

$$\frac{5^{4x} \cdot 5^{12}}{5^{4x}(5^{3} + 5^{-6})}$$

Finally, cancel out the like terms:

$$\frac{5^{12}}{5^{3} + 5^{-6}}$$